STRAND D: Probability

D2 *Probability of Two or More Events*

Text

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D2 Probability of Two or More Events

D2.1 Outcome of Two Events

When dealing with probabilities for two events, it is important to be able to identify all the possible outcomes. Here are examples to show the methods that can be used.



Note *Assume that any dice referred to in this unit is 6-sided and fair. Assume that a coin is fair. A pack of cards comprises 52 cards.*

Method A : Systematic Listing



Worked Example 1

For a special meal, customers at a pizza parlour can choose a pizza with *one* of the following toppings.

Ham
Mushroom
Salami
Pepperoni
Tuna

and a drink from the following list

Cola
Diet Cola
Orange

How many possible combinations of toppings and drinks are there?



Solution

Using the first letter of each drink and topping, it is easy to see that Cola (C)) could be combined with any of the five toppings to give CH, CM, CS, CP, CT. Here 'CH' means 'Cola' drink and 'Ham' topping, etc.

Similarly, for Diet Cola (D), you have

DH, DM, DS, DP, DT

and for Orange (O)

OH, OM, OS, OP, OT

You can see that there are $3 \times 5 = 15$ possible outcomes.

This method of listing will always work but it might be slow, particularly if there are more than 2 choices to be made.

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Method B : 2-way Tables

Worked Example 2

A die and a coin are tossed. List all the possible outcomes.

Solution

The coin can land heads (denoted by H) or tails (T), whilst the die can show 1, 2, 3, 4, 5 or 6. So for heads on the coin, the possible outcomes are

H1, H2, H3, H4, H5 and H6

whilst for tails, they are

T1, T2, T3, T4, T5 and T6.

The listing method used here can be conveniently summarised in a 2-way table.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

This method works well but cannot be used if there are more than 2 choices to be made.

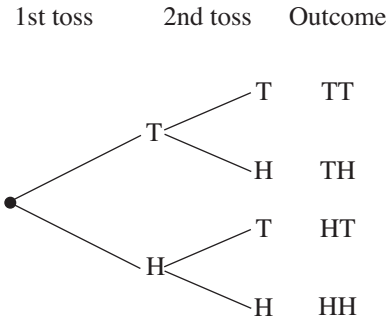
Method C : Tree Diagrams

Worked Example 3

A coin is tossed twice. List all the possible outcomes.

Solution

You can use a tree diagram to represent this solution.



Note that 'TH' is not the same as 'HT'.

This is an excellent method, but can lead to problems if there are too many branches.

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Exercises

1. Two dice are rolled together. Complete the table below to show all the outcomes as total scores.

		Second dice					
		1	2	3	4	5	6
First dice	1						
	2						
	3						
	4						
	5						
	6						

2. Three flavours of ice cream, vanilla (V), mint (M) and raspberry ripple (R), are available at a shop. Each is served with a topping of either chocolate (C) or strawberry (S).

   One possible order is for vanilla ice cream with chocolate topping (VC).

   Write a list of all the other possibilities.

3. A bag contains two balls which are the same size. One is green and one is red. You take a ball out of the bag, put it back, then take another.

   Make a list of all the possible outcomes for the colours of the two balls.

4. Three boys, Ben, John and Nathan, decide to hold a competition in the gym. They will do sit-ups and then press-ups.

   If Ben wins the sit-ups and John wins the press-ups, the outcome would be represented as BJ.

   (a) What does NB represent?

   (b) Make a list of all the 9 possible outcomes.

   (c) If only Ben and John take part in the competition there will be fewer possible outcomes. List the outcomes in this case.

   (d) If Tom also takes part in the competition, list all the possible outcomes for the four competitors.

5. Packets of cereal contain a free model dinosaur. There are four different models, the Brontosaurus (B), the Stagosaurus (S), Tyrannosaurus-Rex (T) and Diplodocus (D). A mother buys two packets of cereal for her children. List all combinations of free gifts possible when the packets are opened.

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6. At a School Fair, three different sorts of prizes can be won in a lucky dip. One is a CD (C), one is a DVD (D) and the other a book (B).

   List all the possible outcomes for a girl who has two trials at the lucky dip.

7. For breakfast, Rachel will drink either fruit juice (F) or cold milk (M) and will eat cornflakes (C), honey-crunch loops (H) or toast (T).

   Complete a copy of the table below to show the possible outcomes for her choice of breakfast.

		Drinks	
		F	M
Food	H		
	T		
	C		

8. List the possible outcomes when 3 coins are tossed.

9. (a) A bag contains 2 red marbles, 1 blue marble and 1 yellow marble.

   A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble. A marble is drawn from each bag.

   Complete the table showing all the possible pairs of colours.

		Marble from second bag			
		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

   (b) 2 marbles are drawn from a third bag.

   The probability that they are both of the same colour is $\frac{5}{9}$.

   What is the probability that they are of different colours?

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D2.2 Probability of Two Events

When two events take place, and every outcome is equally likely to happen, the probability of a particular combined outcome can be readily found from the formula

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

The following examples show how this formula is used.

Worked Example 1

Two dice are thrown together. Find the probability that the total score is 9.

Solution

The table shows all the possible outcomes and total scores.

First dice \ Second dice	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	(9)
4	5	6	7	8	(9)	10
5	6	7	8	(9)	10	11
6	7	8	(9)	10	11	12

There are 36 possible outcomes, and each one is equally likely to occur.

The outcomes that give a total of 9 have been circled. There are 4 such outcomes.

Now the probability can be found.

$$P(9) = \frac{4}{36} = \frac{1}{9}$$

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Worked Example 2

A spinner that forms part of a children's game can point to one of four regions, A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

The table shows all the possible outcomes.

		Second child			
		A	**B**	**C**	**D**
First child	**A**	(AA)	AB	AC	AD
	B	BA	(BB)	BC	BD
	C	CA	CB	(CC)	CD
	D	DA	DB	DC	(DD)

There are 16 possible outcomes. Each is equally likely to occur. The outcomes that are the same for both children have been circled. There are four outcomes of this type.

The probability that both have the same letter is

$$\frac{4}{16} = \frac{1}{4}$$

Note

It is expected that fractions are used for expressing probabilities, but using decimals is equally acceptable.

Exercises

1. When two coins are tossed together the possible outcomes are as shown in the table.

		Second coin	
		H	**T**
First coin	**H**	HH	HT
	T	TH	TT

(a) What is the probability that both coins show heads?

(b) What is the probability that only one coin shows a tail?

(c) What is the probability that both coins land the same way up?

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2. A coin is tossed and a die is rolled. Copy and complete the table below to show the possible outcomes.

		Dice					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1					
	T						

What is the probability of obtaining

(a) a head and a 6,

(b) a tail and an odd number,

(c) a head and an even number,

(d) a head and a number greater than 2,

(e) an even number?

3. (a) Use this table, which shows the outcomes when two dice are rolled, to find the probabilities of each event described below.

		Second dice					
		1	**2**	**3**	**4**	**5**	**6**
First dice	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(i) A score of 7.

(ii) A score of 5.

(iii) A score that is an even number.

(iv) A score of more than 8.

(v) A score of less than 6.

(b) What score are you most likely to get when you roll *two* dice?

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4. A school cook decides at random what flavour drink to serve at lunch time. She chooses from blackcurrant (B), orange (O) and lemon (L).

(a) Complete a copy of the table to show the possible outcomes from two consecutive days.

(b) What is the probability that she serves:
(i) blackcurrant on both days,
(ii) the same flavour on both days,
(iii) lemon or blackcurrant on both days?

		Second day		
		B	**L**	**O**
First day	**B**	BB		
	L			LO
	O			

(c) Clare will not drink lemon squash. What is the probability that she is unable to drink the squash on two consecutive days?

5. A young couple decide that they will have two children. There is an equal chance that each child will be a boy or a girl.

(a) Find the probability that both children are boys.

(b) Find the probability that both children are of the same sex.

6. A game contains two tetrahedral dice which have 4 faces numbered 1 to 4. The two dice are thrown, and the total score is noted.

(a) Find the probability that a score of 3 is obtained.

(b) Find the probability of getting a score of more than 4.

(c) Which score is most likely?

7. A bag contains one red ball, one blue ball and one green ball. One ball is taken out of the bag. A second ball is also taken out, without replacing the first ball. The table shows the possible outcomes.

(a) Explain why some entries in the table have been marked with an X. How many possible outcomes are there?

(b) What is the probability that the red ball is selected?

(c) What is the probability that the green ball is left in the bag?

		Second ball		
		R	**B**	**G**
First ball	**R**	X	RB	RG
	B	BR	X	BG
	G	GR	GB	X

8. Three coins are tossed at the same time. Find the probabilities that

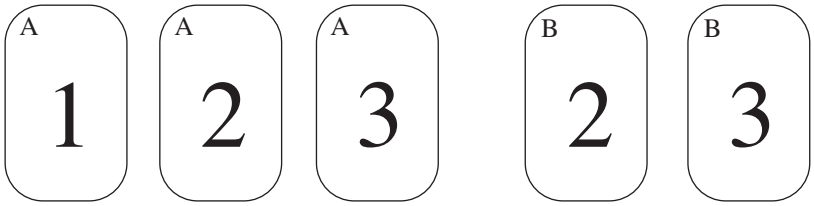
(a) they all land the same way up,

(b) they all land with heads showing,

(c) at least one coin lands showing tails.

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9. The diagram shows two sets of cards A and B.



(a) One card is chosen at random from set A. One card is chosen at random from set B.

(i) List all the possible outcomes.

The two numbers are added together.

(ii) What is the probability of getting a total of 5?

(iii) What is the probability of getting a total that is **not** 5?

A new card is added to the set B. It is



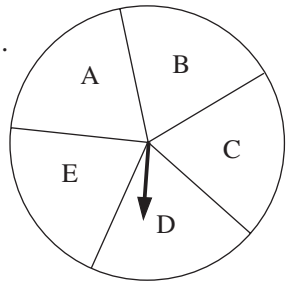
One card is chosen at random from set A. One card is chosen at random from the new set B.

(b) (i) How many possible outcomes are there now?

(ii) Explain why adding the new card *does not* change the number of outcomes that have a total of 5.

(iii) Explain why adding the new card *does* change the probability of getting a total of 5.

10. The first diagram shows an unbiased spinner used in a game.

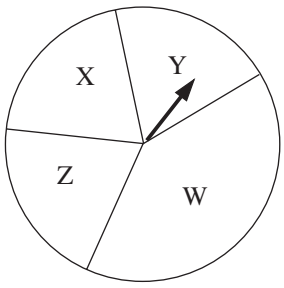


It is divided into five equal sections.

The arrow is spun once.

(a) What is the probability that the arrow will land on section *A*?

The second diagram shows another unbiased spinner.



Section W is twice the size of each of the other sections.

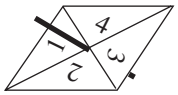
(b) What is the probability that the arrow on this spinner will land on section W?

(c) When both arrows are spun at once, one outcome could be AW.

List the other possible outcomes.

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11. A four sided spinner is spun and a die is rolled.





The two scores are then multiplied to give a result.

(a) Complete the table to show all the possible results.

Spinner \ Die	1	2	3	4	5	6
1						
2						
3						
4						

(b) The spinner is spun once and the die is rolled once.

What is the probability of getting a result of 12?

12. (a) A bag contains 2 red marbles, 1 blue marble and 1 yellow marble. A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble.

A marble is drawn from each bag.

Complete the table below, showing all the possible pairs of colours.

Marble from first bag \ Marble from second bag	R	B	B	Y
R	RR	RB	RB	RY
R	RR			
B	BR			
Y	YR			

(b) 2 marbles are drawn from a third bag. The probability that they are both of the same colour is $\frac{5}{9}$.

What is the probability that they are of different colours?

D2.3 Use of Tree Diagrams

Tree diagrams can be used to find the probabilities for two events, when the outcomes are not necessarily equally likely.

Worked Example 1

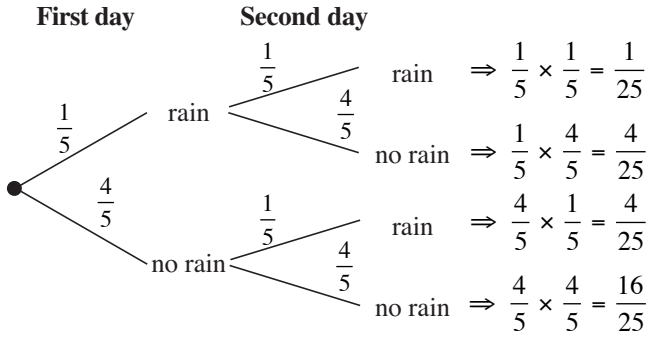
If the probability that it rains on any day is $\frac{1}{5}$, draw a tree diagram and find the probability

(a) that it rains on two consecutive days,

(b) that it rains on only one of two consecutive days.

Solution

The tree diagram shows all the possible outcomes. Then the probability of each event can be placed on the appropriate branch of the tree. The probability of no rain is $1-\frac{1}{5}=\frac{4}{5}$.



The probability of each outcome is obtained by multiplying together the probabilities on the branches leading to that outcome. For rain on the first day, but not on the second, the probability is

$$\frac{1}{5}\times\frac{4}{5}=\frac{4}{25}$$

(a) The probability that it rains on two consecutive days is given by the top set of branches, and is $\frac{1}{25}$.

(b) There are two outcomes where there is rain on only one of the two days. These are rain – no-rain, with a probability of $\frac{4}{25}$ and no-rain – rain with a probability of $\frac{4}{25}$.

The probability of rain on only one day is found by adding these two probabilities together:

$$\frac{4}{25}+\frac{4}{25}=\frac{8}{25}$$

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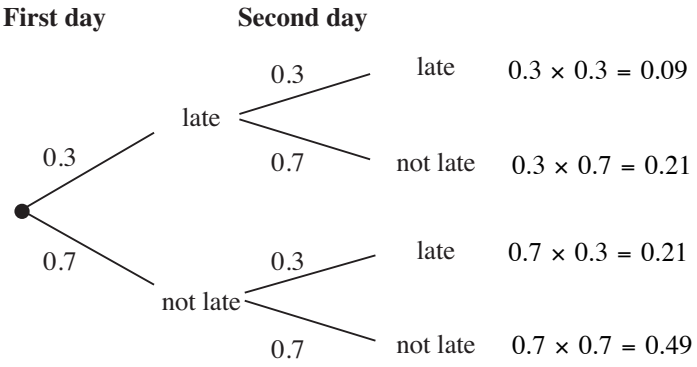
Worked Example 2

The probability that Jenny is late for school is 0.3. Find the probability that on two consecutive days she is:

(a) never late, (b) late only once.

Solution

The tree diagram shows the possible outcomes and their probabilities. Note that the probability of not being late is $1 - 0.3 = 0.7$.



The probabilities on each set of branches are multiplied together to give the probability of that outcome.

(a) The probability that Jenny is never late is given by the bottom set of branches and has probability 0.49.

(b) The probability that she is late once is given by the two middle sets of branches which both have a probability 0.21. So the probability that she is late once is given by

$$0.21 + 0.21 = 0.42$$

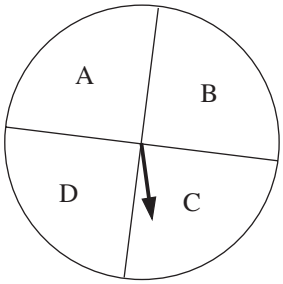
Note

The method shown here also works for problems when the outcomes are equally likely (as in the previous method) – it is sometimes rather cumbersome though to draw all the branches.

The next example is the same as Worked Example 2 in Section D2.2, but this time the tree diagram method will be used.

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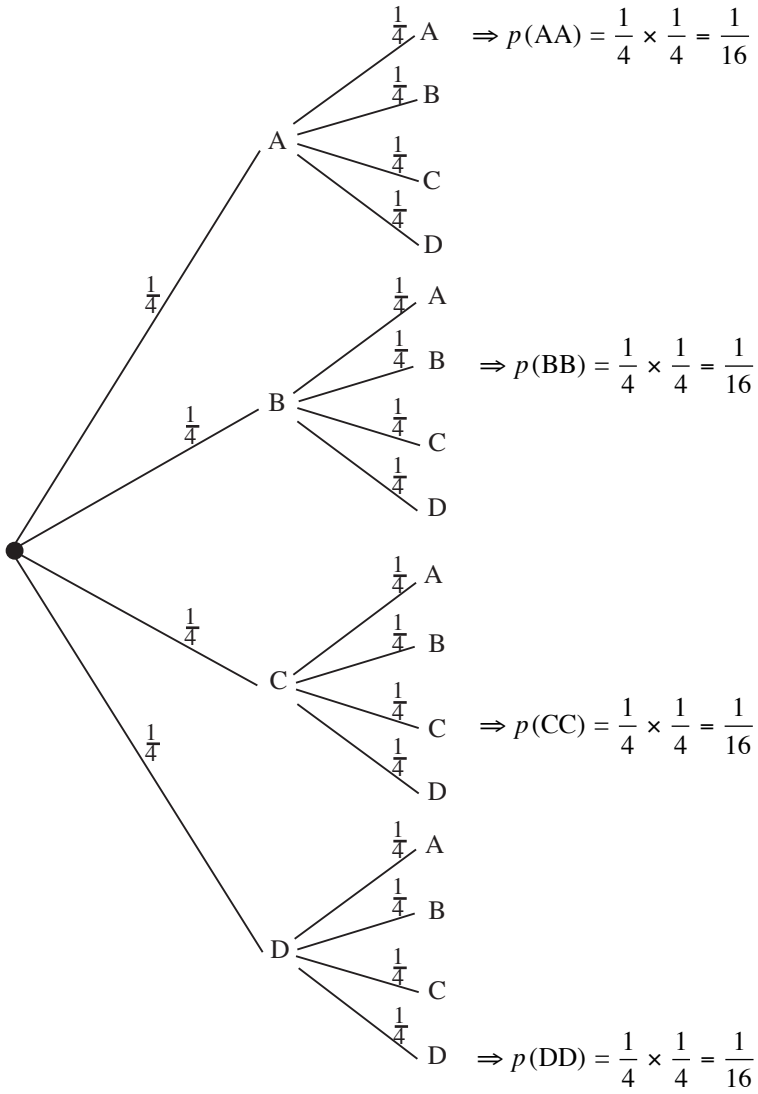
Worked Example 3



A spinner that forms part of a children's game can point to one of four regions, A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

This time, let us use the tree diagram approach.



So the probability of both children obtaining the same letter is

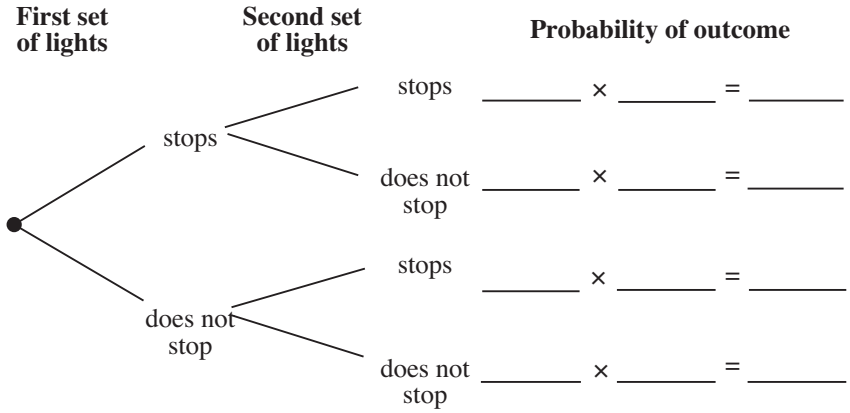
$$\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{1}{4} \text{ (as obtained before)}$$

Exercises

1. On a route to a factory a truck must pass through two sets of traffic lights. The probability that the truck has to stop at a set of lights is 0.6.

   (a) What is the probability that the truck does not have to stop at a set of traffic lights?

   (b) Copy the tree diagram below and add the correct probabilities to each branch.



   (c) What is the probability that the truck gets to the factory without having to stop at a traffic light?

   (d) What is the probability that the truck stops at both sets of traffic lights?

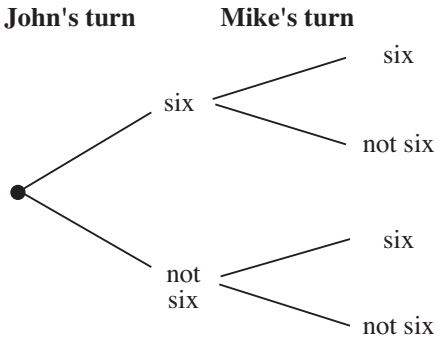
   (e) What is the probability that the truck stops at one set of traffic lights?

2. Two boys are playing a game. They take it in turns to start. Before they start they must throw a six. John starts first.

   (a) What is the probability of throwing a six?

   (b) Copy the tree diagram and add the appropriate probabilities to each branch.

   Also calculate the probability of each outcome shown on the tree diagram.



   (c) Find the probability that:

   (i) both boys start the game on their first throws,

   (ii) only one of them starts the game on their first throw,

   (iii) neither of them starts the game on their first throw.

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3. Draw a tree diagram to show the possible outcomes when two coins are tossed. Include the probabilities on your tree diagram.

   Find the probability of obtaining:

   (a) two heads, (b) no heads, (c) only one head.

4. Mike travels to Swindon from Bristol on the early bus. The probability that he arrives late is $\frac{1}{10}$. He catches the bus on two consecutive days.

   What is the probability that he arrives:

   (a) on time on both days,

   (b) on time on at least one day,

   (c) late on both days.

5. When Sheila's office phone rings the probability that the call is for her is $\frac{3}{4}$.

   (a) What is the probability that a call is not for Sheila?

   (b) Draw a tree diagram that includes probabilities to show the possible outcomes when the phone rings twice.

   (c) Find the probabilities that:

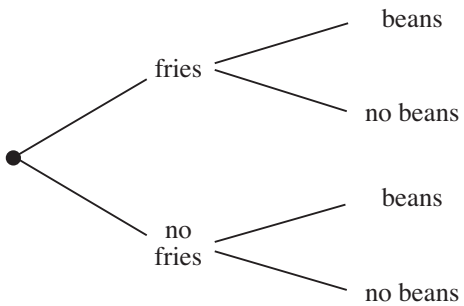
   (i) both calls are for Sheila,

   (ii) only one call is for Sheila,

   (iii) neither call is for Sheila.

6. In a school canteen the probability that a student has fries with their meal is 0.9 and the probability that they have beans is 0.6.

   (a) Copy and complete the tree diagram below.



   (b) What is the probability that a student has:

   (i) both fries and beans,

   (ii) fries but not beans,

   (iii) neither fries nor beans?

7. To be able to drive a car unsupervised you must pass both a theory test and a practical driving test. The probability of passing the theory test is 0.8 and the probability of passing the practical test is 0.6.

   (a) What is the probability of failing:

   (i) the theory test,

   (ii) the practical test?

   (b) What is the probability that someone:

   (i) passes both tests,

   (ii) fails both tests?

8. Matthew and Adam play chess together. The probability that Adam wins is 0.52.

   (a) Find the probabilities that, out of two games,

   (i) Adam wins two,

   (ii) Matthew wins two,

   (iii) they win one each.

   (b) Which of the outcomes is the most likely?

9. Victoria calls for her friends, Kina and Freya. The probability that Kina is not ready to leave is 0.2 and the probability that Freya is not ready is 0.3.

   Use suitable tree diagrams to find the probability that:

   (a) both Freya and Kina are ready to leave,

   (b) one of them is not ready to leave,

   (c) Kina is not ready to leave on two successive days,

   (d) Kina is ready to leave on two consecutive days.

10. A die has 6 faces of which 3 are green, 2 yellow and 1 red.

    Find the probabilities of the following outcomes if the die is rolled twice.

    (a) Both faces have the same colour.

    (b) Both faces are red.

    (c) Neither face is green.

11. (a) Draw a tree diagram to show the possible outcomes when a coin is tossed three times.

    (b) Find the probability of obtaining:

    (i) 3 heads or 3 tails,

    (ii) at least 2 heads,

    (iii) exactly one tail.

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12. On average, Anna comes to dinner on 2 days out every 5. If Anna comes to dinner, the probability that we have ice cream is 0.7.

    If Anna does not come to dinner, the probability that we have ice cream is 0.4.

    (a) Draw a tree diagram to illustrate this information.

    Write the appropriate probability on each branch.

    (b) What is the probability that we will have ice cream at dinner tomorrow?

13. Mervin has 8 red socks and 6 white socks all mixed up in his sock drawer.
    He takes 2 socks at random from the drawer.

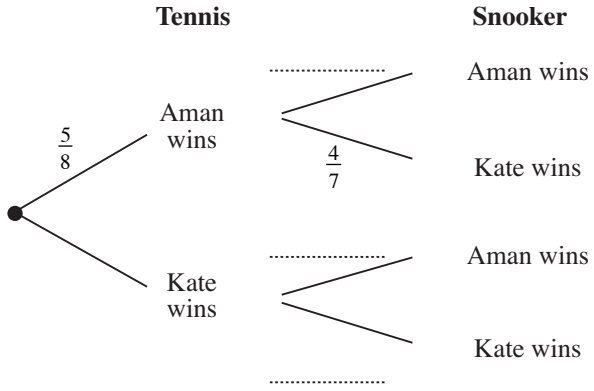
    (a) If the first sock that Mervin takes is red, what is the probability that the second sock will also be red?

    (b) What is the probability that Mervin will take 2 socks of the same colour?

14. Aman and Kate play a game of tennis. The probability that Aman will win is $\frac{5}{8}$.

    Aman and Kate play a game of snooker. The probability that Kate will win is $\frac{4}{7}$.

    (a) Copy and complete the probability tree diagram below.



    (b) Calculate the probability that Kate will win both games.

15. A die is biased as follows:

    The probability of scoring a 6 is 0.4. The probability of scoring a 5 is 0.2.

    (a) Jessica throws the die once. Calculate the probability that the score will be 5 or 6.

    (b) Joseph throws the die twice. Calculate the probability that both scores will be 6s.

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16. A fair die is thrown three times.

    (a) What is the probability of throwing 3 sixes?

    (b) What is the probability of throwing a six on the first throw, a six on the second throw but not a six on the third throw?

    (c) What is the probability of throwing exactly two sixes in the three throws?

    (d) What is the probability of throwing at least two sixes in the three throws?

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