STRAND D: PROBABILITY

Unit D3 *Extending Probability*

Text

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D3 Data Collection

D3.1 Multiplication for Independent Events

Two events are *independent* if one event happening does not affect the probability of the other event. In this case the probability of two events A and B occurring is given by

$$p(\text{A and B}) = p(\text{A}) \times p(\text{B})$$

Worked Example 1

A die is rolled twice. If event A is the first roll shows a six and event B is the second roll shows a six,

(a) are events A and B independent?

(b) Find p(A and B).

Solution

(a) The events are independent as the number obtained on the first roll does not affect the second roll.

(b) $$p(\text{A}) = \frac{1}{6} \text{ and } p(\text{B}) = \frac{1}{6}$$

so $$p(\text{A and B}) = p(\text{A}) \times p(\text{B})$$
$$= \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

Worked Example 2

A spinner in game has 3 sections of equal size that are coloured red, blue and green. Let the events B, R and G be:

B: the spinner lands a blue

G: the spinner lands on green

R: the spinner lands on red.

(a) Are these events independent?

(b) Find the probability that when the spinner is spun twice the following outcomes are obtained.

(i) Red both times.

(ii) Red and green in any order.

(iii) Both are the same colour.

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Solution

(a) The events are independent as the result of one spin does not affect the next.

(b) The probabilities of each event are

$$p(\mathrm{B}) = \frac{1}{3} \quad p(\mathrm{R}) = \frac{1}{3} \quad p(\mathrm{G}) = \frac{1}{3}.$$

(i) The probability is given by $p(\mathrm{R \text{ and } R}) = p(\mathrm{R}) \times p(\mathrm{R})$

$$= \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{9}$$

(ii) For red and green in any order, two outcomes must be considered: Red then Green and Green then Red.

$$p(\mathrm{G \text{ and } R}) = p(\mathrm{G}) \times p(\mathrm{R}) \qquad p(\mathrm{R \text{ and } G}) = p(\mathrm{R}) \times p(\mathrm{G})$$
$$= \frac{1}{3} \times \frac{1}{3} \qquad = \frac{1}{3} \times \frac{1}{3}$$
$$= \frac{1}{9} \qquad = \frac{1}{9}$$

Hence the probability of a red and green in any order is given by:

$$p(\mathrm{R \text{ and } G}) + p(\mathrm{G \text{ and } R}) = \frac{1}{9} + \frac{1}{9}$$
$$= \frac{2}{9}$$

(iii) For both to be the same colour the outcomes, R and R, G and G, B and B must be considered.

From (i) $p(\mathrm{R \text{ and } R}) = \frac{1}{9}$

Similarly $p(\mathrm{B \text{ and } B}) = \frac{1}{9}$

and $p(\mathrm{G \text{ and } G}) = \frac{1}{9}$

The probability that both spins are the same colour is given by:

$$p(\mathrm{R \text{ and } R}) + p(\mathrm{B \text{ and } B}) + p(\mathrm{G \text{ and } G}) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

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Exercises

1. For each pair of events A and B listed below, decide whether or not it is likely that the events are independent.

   (a) A: It rains today.

   B: It rains tomorrow.

   (b) A: It rains on Monday this week.

   B: It rains on Monday next week.

   (c) A die is rolled twice.

   A: The first roll shows a 3.

   B: The second roll shows a 5.

   (d) A baby is born.

   A: Its left eye is blue.

   B: Its right eye is blue.

   (e) Joshua and James are brothers.

   A: Joshua catches measles.

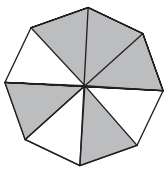
   B: James catches measles.

   (f) Daniel cycles to school.

   A: Daniel's bicycle has a puncture.

   B: Daniel is late for school.

2. The spinner shown in the diagram has eight sections of equal size; each one is coloured white or black.



   The events B and W are:

   B: the spinner lands on black,

   W: the spinner lands on white.

   (a) Find the following probabilities:

   (i) p(B) (ii) p(W) (iii) p(B and B)

   (iv) p(W and W) (v) p(B and W) (vi) p(W and B).

   (b) If the spinner is spun twice find the probabilities of the following outcomes.

   (i) White is obtained both times.

   (ii) A different colour is obtained on each spin.

   (iii) The same colour is obtained on each spin.

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3. A bag contains 7 red balls and 3 green balls. A ball is taken out and replaced. A second ball is then taken out.

   R is the event that a Red ball is selected. G is the event that a Green ball is selected.

   (a) Find the following probabilities

   (i) p(R) (ii) p(G) (iii) p(G and G)

   (iv) p(R and R) (v) p(G and R) (vi) p(R and G).

   (b) Find the probability that if two balls are taken in turn;

   (i) they are both red,

   (ii) they are different colours,

   (iii) they are the same colour.

4. A die is thrown twice. Find the probability that

   (a) two odd numbers are obtained,

   (b) the same two numbers are obtained.

5. The probability that Nigel is late for work is 0.2. The probability that Karen is late for work is 0.3. Assume that these events are independent.

   (a) Find the probability that they are both late for work.

   (b) Would your answer be the same if you knew that Karen was going to give Nigel a lift in her car?

6. In a school 20% of the children are colour blind and 10% are left handed. If a child is selected at random, what are the probabilities that they are:

   (a) neither colour blind or left handed,

   (b) colour blind and left handed,

   (c) left handed but not colour blind.

7. Assume that 50% of the population of Britain are women, that 20% of the population are vegetarians and that men and women are equally likely to be vegetarians. Find the probability that a person selected at random is:

   (a) a male vegetarian,

   (b) a meat-eating female,

   (c) a meat-eating male,

   (d) a female vegetarian.

8. Ben's mum always gives him cheese or jam sandwiches in his packed lunch. The probability of her giving him cheese sandwiches is 0.3. She also gives him a chocolate biscuit or a wafer bar. The probability that she gives him a chocolate biscuit is 0.4. Find the probability that Ben's packed lunch contains:

   (a) a wafer bar and jam sandwiches,

   (b) a chocolate biscuit on two consecutive days.

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9. When Wendy plays Tetrix on her computer, the probability that she scores more than 1000 points is $\frac{1}{7}$. One day Wendy plays two games of Tetrix.

   (a) What is the probability that she scores over 1000 in both games?

   (b) What is the probability that she scores less than 1000 in both games?

   (c) What is the probability that she scores over 1000 in one of the two games?

   (d) If Wendy plays three games instead of two, what is the probability that she scores over 1000 in all three games?

10. Once a week John checks his car. The probability that he needs to pump up a tyre is $\frac{1}{20}$. The probability he has to add oil is $\frac{1}{10}$ and the probability he has to add water is $\frac{1}{5}$.

   (a) Is it reasonable to assume that the events described above are independent?

   For questions (b) and (c), assume that the events are independent.

   (b) What is the probability that John does not need to do anything to his car?

   (c) What is the probability that John has to do one thing to his car, that is blow up a tyre, add oil or add water?

11. The probability that a woman in Mathsville is a least 165 cm tall is 0.15.

   The probability that a woman in Mathsville is coloured-blind is 0.02.

   These probabilities are independent of each other.

   (a) What is the probability that a woman in Mathsville is both colour-blind and at least 165 cm tall?

   (b) What is the probability that a woman in Mathsville is less than 165 cm tall and is not colour-blind?

12. Tom cycles to school on Fridays. There is a probability of $\frac{1}{5}$ that he will be late and a probability of $\frac{4}{5}$ that he will not be late.

   (a) Calculate the probability that Tom will be late on two consecutive Fridays.

   (b) Using *L* for 'late' and *N* for 'not late', complete the table to show the possible outcomes of Tom's journeys for the two Fridays.

1st Friday	2nd Friday
L	*L*

   Hence calculate the probability that Tom will be late on at least one of the two Fridays.

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13. A student takes examinations in Mathematics, English and French.

The probability that she passes Mathematics is 0.7.

The probability that she passes English is 0.8.

The probability that she passes French is 0.6.

Given that her results in each subject are independent, find the probability that

(a) she fails Mathematics or French or both;

(b) she fails English or French but *not* both.

14. In the game of 'Pass the Pig', two identical toy pigs are thrown. Each pig can land in one of five positions. The five positions and the probabilities that the pig will land in each of these positions are shown in the table.

Position	Sider	Trotter	Razorback	Snouter	Leaning Jowler
					
Probability	0.57	0.2	0.2	0.02	0.01

Both pigs are thrown.

Work out the probability that they will both land in the 'Trotter' position.

D3.2 Mutually Exclusive Events

If two events cannot happen or take place at the same time, then they are called *mutually exclusive* events. For example when tossing a single coin the events 'heads' and 'tails' are mutually exclusive because they cannot both be obtained at the same time.

If A and B are mutually exclusive events then the probability of obtaining A or B is given by:

$$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$

Worked Example 1

State whether or not the pairs of events described below are mutually exclusive.

(a) A: A die is rolled and shows a 6.
B: A die is rolled and shows an odd number.

(b) A: Selecting a child with blue eyes from a class.
B: Selecting a left handed person from a class.

Solution

(a) It is not possible for a die to show a six and an odd number at the same time, so these events are mutually exclusive.

(b) It is possible to select a person who has blue eyes and is left handed, so these events are not mutually exclusive.

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Worked Example 2

When Andrew buys a can of drink the probabilities of selecting particular brands are given in the table below:

Drink	Probability
Cola	$\frac{1}{2}$
Lemonade	$\frac{1}{9}$
Fizzo	$\frac{1}{4}$

Find the probabilities that he selects:

(a) Cola or Lemonade

(b) Fizzo or Lemonade

(c) none of the drinks listed above.

Solution

(a) $p(\text{Cola or Lemonade}) = p(\text{Cola}) + p(\text{Lemonade})$

$$= \frac{1}{2} + \frac{1}{9} = \frac{9}{18} + \frac{2}{18}$$

$$= \frac{11}{18}$$

(b) $p(\text{Fizzo or Lemonade}) = p(\text{Fizzo}) + p(\text{Lemonade})$

$$= \frac{1}{4} + \frac{1}{9}$$

$$= \frac{9}{36} + \frac{4}{36}$$

$$= \frac{13}{36}$$

(c) First find the probability that he chooses one of the drinks from the list.

$p(\text{Cola or Lemonade or Fizzo}) = p(\text{Cola}) + p(\text{Lemonade}) + p(\text{Fizzo})$

$$= \frac{1}{2} + \frac{1}{9} + \frac{1}{4}$$

$$= \frac{18}{36} + \frac{4}{36} + \frac{9}{36}$$

$$= \frac{31}{36}$$

then

$p(\text{drink not from list}) = 1 - p(\text{drink from list})$

$$= 1 - \frac{31}{36}$$

$$= \frac{5}{36}$$

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Exercises

1. When a die is rolled, the following outcomes can be used to describe the result.

   A: An odd number is obtained.

   B: An even number is obtained.

   C: A prime number is obtained.

   Copy and complete the following statements.

   A and ___ are mutually exclusive.

   B and ___ are not mutually exclusive.

2. Decide if each pair of events A and B given below are mutually exclusive.

   (a) A: Winning a football match.
   B: Losing a football match.

   (b) A: Selecting a diamond from a pack of playing cards.
   B: Selecting a King from a pack of playing cards.

   (c) A: Arriving late on a train journey.
   B: Not arriving early on a train.

   (d) A: Selecting an ace from a pack of cards.
   B: Selecting a queen from a pack of cards.

   (e) A: It rains tomorrow.
   B: It is sunny tomorrow.

3. When Plymouth Argyle football team play in a league match the probability that they win is 0.4 and the probability that they draw is 0.3. What is the probability that they lose?

4. When Samantha, Annie and Katie play a game, the probability that Samantha wins is $\frac{1}{2}$ and the probability that Annie wins is $\frac{1}{3}$. What is the probability that Katie wins?

5. A bag contains a number of balls, which are yellow, blue or green. The probability of selecting a ball at random and getting a green is $\frac{1}{7}$ and the probability of getting a yellow is $\frac{3}{7}$.

   (a) What is the probability of getting a blue ball?

   (b) If the bag contains 4 green balls, how many yellow balls does it contain?

   (c) If the bag contains 6 blue balls, how many balls does the bag contain in total?

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6. When a car arrives at a set of traffic lights the probability that a green light is showing is $\frac{7}{20}$ and the probability that a red light is showing is $\frac{3}{10}$.

   (a) What is the probability that neither the green or red lights are showing?

   (b) Explain why the probability that an amber light is showing, is greater than your answer to (a).

7. A bag contains 6 red counters, 5 blue counters and 4 pink counters. A counter is selected from the bag at random.

   Find, the probability that the counter is:

   (a) either red or pink, (b) not pink,

   (c) not red, (d) blue or pink.

8. A bag contains a number of balls of different colours. The probability of obtaining a ball of a particular colour is given in the table below.

Colour	**Probability**
Red	$\frac{3}{8}$
Green	$\frac{1}{4}$
Blue	$\frac{1}{5}$

   What is the probability that a ball taken from the bag is:

   (a) red or green,

   (b) not blue or green,

   (c) not one of the colours listed above?

9. As part of a project, a group of students classify each day as *Dry*, *Damp*, *Wet* or *Very wet*. The probability that a day will be *Dry* or *Damp* is 0.6. The probability that a day is *Wet* is 0.3.

   (a) What is the probability that a day is *Very wet*?

   The probability that the day is *Damp* is twice the probability that it is *Dry*.

   (b) What is the probability that it is *Damp* or *Wet*?

10. A pack contains cards that are coloured pink, yellow or black. When a card is chosen at random the probability of obtaining a black or pink card is $\frac{5}{7}$ and the probability of obtaining a black or yellow card is $\frac{3}{5}$.

    Find the probability of obtaining a card of each colour.

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11. In a class of 25 students there are 8 students with size 6 feet, 4 students with size 7 feet and 3 students that are left handed.

Find:

(a) the probability that a student in this class has size 6 or size 7 feet,

(b) the probability that a student in this class is right-handed.

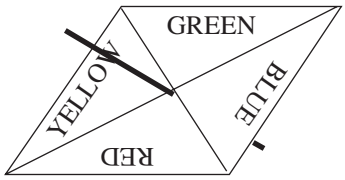
Is it possible to find the probability that:

(c) a student has size 6 feet and is left-handed.,

(d) has feet that are bigger than size 7,

(e) has feet that are not size 6 or size 7?

12. The spinner shown is biased.



The probabilities of getting a particular colour are shown in the table below:

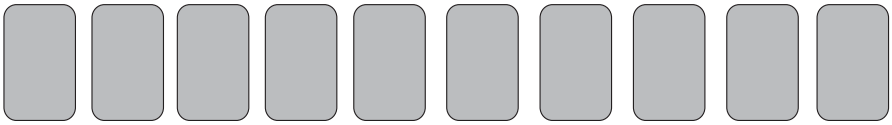
(a) Complete the table to show the probability of getting *Green*.

Colour	*Red*	*Yellow*	*Blue*	*Green*
Probability	0.4	0.1	0.3	

(b) The spinner is spun once. What is the probability of getting either *Red* or *Blue*?

(c) The spinner is spun 50 times. Approximately how many times would you expect to get *Red*?

13. A pack of 52 playing cards consists of equal numbers of clubs, diamonds, hearts and spades. Ten cards are removed from the pack and placed face down on a table.



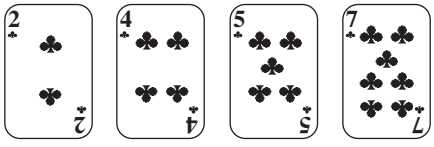
When one of these cards is taken at random the following probabilities apply:

Type of card	**Probability**
club	0.4
heart	0.2
spade	0.1
diamond	0.3

Number on card	**Probability**
2	0.2
3	0.2
4	0.1
5	0.3
7	0.2

Four of the ten cards are clubs.

They are numbered 2, 4, 5 and 7.



One of the ten cards is taken from the table at random.

(a) What is the probability that it is *not* a diamond?

(b) What is the probability that it is a club or a diamond?

(c) What is the probability that it is a club or numbered 3?

(d) Explain why the probability that it is a club or numbered 5 is *not* 0.4 + 0.3.

14. A bag contains a total of 20 beads. There are 6 red beads, 9 blue beads and 5 white beads.

(a) A bead is taken at random from the bag. The probability that it is red is 0.3.

(i) What is the probability that it is white?

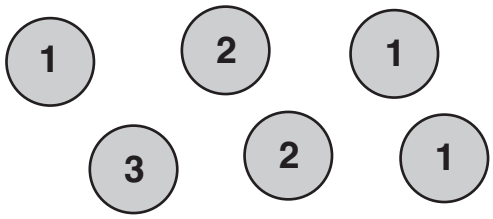
(ii) What is the probability that it is **not** white.

All the beads are taken from the bag, numbered 1, 2, 3 or 4 and then replaced.

When a bead is taken from the bag at random the probability of each number is as shown in the table.

Number on bead	Probability
1	0.3
2	0.4
3	0.2
4	0.1

The red beads are numbered,



(b) A bead is taken at random from the bag.

(i) What is the probability that it is red or numbered 4?

(ii) Explain why the probability of getting a red bead or a bead numbered 2 is *not* 0.3 + 0.4.

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D3.3 Tree Diagrams and Conditional Probability

The probabilities of certain events may change as a result of earlier events. For example, if it rains today, the probability that it rains tomorrow may be greater than if it was sunny today. When using a tree diagram, check that the probabilities do not change as a result of the first event. When probabilities depend on previous events they are called *conditional probabilities*.

Worked Example 1

A bag contains 7 red balls and 3 green balls. Two balls are taken from the bag. Find the probabilities that they are:

(a) both red (b) both green (c) one red and one green.

Solution

For the first ball taken from the bag,

$$p(\text{Red}) = \frac{7}{10} \text{ and } p(\text{Green}) = \frac{3}{10}$$

If the first ball was red the bag now contains:

6 red balls and 3 green balls

so $p(\text{Red}) = \frac{6}{9}$ and $p(\text{Green}) = \frac{3}{9}$

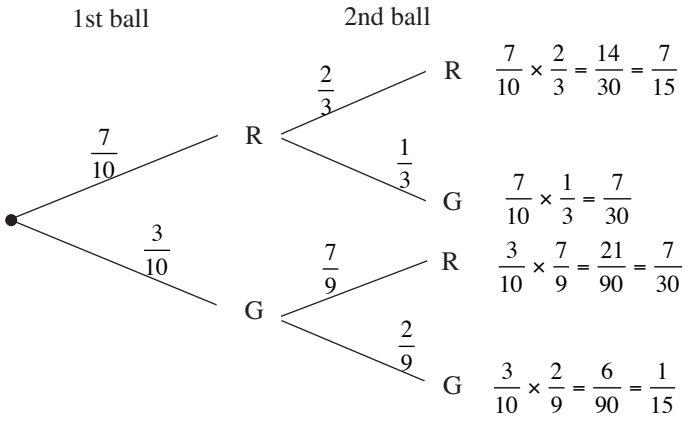
$= \frac{2}{3}$ $= \frac{1}{3}$

If the first ball was green the bag now contains:

7 red balls and 2 green balls

so $p(\text{Red}) = \frac{7}{9}$ and $p(\text{Green}) = \frac{2}{9}$

These probabilities can be used to create a tree diagram.



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So
$$p(\text{both red}) = \frac{7}{15}$$
$$p(\text{both green}) = \frac{1}{15}$$
$$p(\text{one red and one green}) = \frac{7}{30} + \frac{7}{30}$$
$$= \frac{14}{30}$$
$$= \frac{7}{15}.$$

Worked Example 2

Weather experts estimate the probability of rain any day as

0.6 if it rained the previous day

0.3 if it did not rain the previous day.

Find the probability that a dry day is followed by,

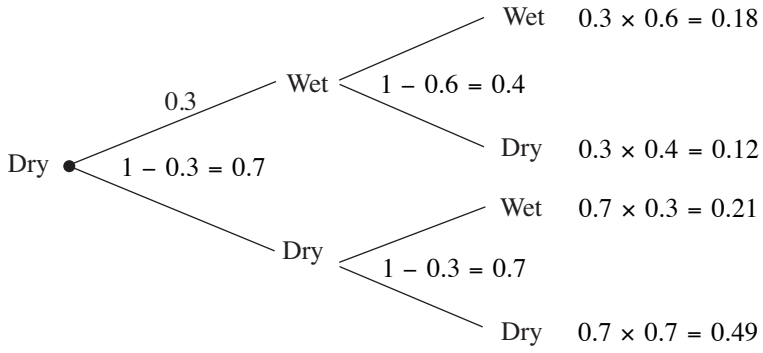
(a) two more dry days,

(b) two wet days,

(c) a wet day and then a dry day.

Solution

The tree diagram has the probabilities included on its branches. Note how they depend on the weather on the previous day.



p(two more dry days) $= 0.49$

p(two wet days after the dry day) $= 0.18$

p(dry followed by a wet day and then a dry day) $= 0.12$

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Exercises

1. A bag contains 3 red counters and 4 blue counters. Two counters are taken out of the bag. Find the probability that:

   (a) both counters are red,

   (b) both counters are the same colour,

   (c) the counters are different colours.

2. A card is drawn at random from a full pack of playing cards. It is not put back. A second card is then drawn from the same pack.

   (a) Find the probability that:

      (i) both cards are hearts,

      (ii) only one card is a heart,

      (iii) neither card is a heart.

   (b) Find the probability that:

      (i) both cards are sevens,

      (ii) neither card is a seven,

      (iii) at least one seven is obtained.

3. A drawer contains 6 white socks and 8 black socks. Two socks are taken from the drawer at random. What is the probability that a pair is obtained (i.e. both white or both black)?

4. The probability that a football team win their next match is:

   $$\begin{cases} 0.3 & \text{if they won their last match,} \\ 0.8 & \text{if they lost their last match.} \end{cases}$$

   (a) Find the probability that if a match is lost, the next two will be won.

   (b) Find the probability that if a match is won, the next two will also be won.

5. Buses arriving at a depot are more likely to be late if the last bus to arrive was also late. The probability that a bus is late is:

   $$\begin{cases} 0.9 & \text{if the previous bus was on time,} \\ 0.4 & \text{if the previous bus was late.} \end{cases}$$

   (a) If a bus arrives late, find the probability that:

      (i) the next two buses will be late,

      (ii) the next two buses will be on time,

      (iii) one of the next two buses will be late.

   (b) If a bus arrives on time, find the probability that the next two buses will also be on time.

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6. A bag contains 3 red balls, 3 green balls and 3 yellow balls. Two balls are taken out of the bag at random. What is the probability that:

   (a) both balls are yellow,

   (b) both balls are the same colour,

   (c) both balls are different colours.

7. A pack of 24 cards contains 12 red cars, 8 yellow cards and 4 pink cards. In a game a player has to select two cards, one after the another. Find the probabilities that a player selects,

   (a) 2 red cards, (b) 2 yellow cards,

   (c) 2 pink cards, (d) 2 cards that are the same colour,

   (e) a pink card and a yellow card in any order.

8. Nisha buys a bag of sweets. The bag contains 3 orange, 17 strawberry and 10 cherry flavour sweets. Nisha eats 3 sweets.. Find the probability that she has:

   (a) not eaten an orange sweet,

   (b) eaten 3 sweets of the same flavour,

   (c) only eaten strawberry flavour sweets.

9. Two yachts A and B regularly race against each other. In rough weather the probability that A wins is 0.8 and in fine weather 0.4. The probability of fine weather at a race is 0.7.

   What is the probability that A wins a race?

   If there is a best of 3 races competition, what is the probability that A wins the competition?

10. Julie always buys a pasty for lunch. She is equally likely to choose any pasty, but never has the same type of pasty on two consecutive days. She never eats pasties at the weekend. The pasties on offer at the shop are:

    Vegetable *Chicken* *Steak* *Tuna*.

    What is the probability that:

    (a) she eats a vegetable pasty on Tuesday,

    (b) she does not eat a chicken pasty on Monday or Tuesday,

    (c) she eats a pasty containing meat on both Monday and Tuesday?

    What is the probability that she has not had a tuna pasty after 5 days?

11. Brian enjoys swimming. If it is a sunny day the probability that he swims is 0.9. It the day is not sunny, the probability that he swims is 0.65.

    The probability that tomorrow will be sunny is 0.8.

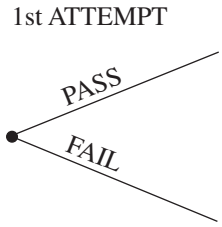
    (a) Draw a tree diagram to illustrate this information.

    (b) Calculate the probability that Brian will not swim tomorrow.

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12. Alex takes a driving test. The probability that he will pass on his first attempt is 0.4. If he fails his first test then the probability that he will pass the test on any subsequent attempt is 0.7.

(a) Copy and complete the tree diagram to show all the possible outcomes for his first two attempts.



(b) Calculate the probability that Alex needs two attempts, and passes on his second attempt.

(c) Calculate the probability that he passes the test after three or four attempts.

13. Wes gives Bronwen a box of 25 mixed sweets. 12 of the sweets have soft centres, 8 have toffee centres and 5 have hard centres. All of the sweets have identical wrappers. Bronwen chooses 2 sweets at random.

(a) What is the probability that Bronwen will choose 2 toffee centred sweets?

After they have eaten 3 soft centred sweets and 2 toffee centred sweets, Wes chooses 3 sweets at random from the 20 sweets that are left.

(b) What is the probability that Wes will choose a soft centred sweet, followed by a toffee centred sweet, followed by a hard centred sweet?

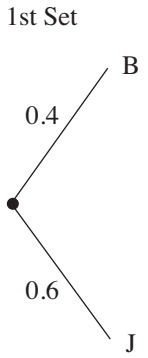
When there are only 10 sweets left, 3 have soft centres, 4 have toffee centres and 3 have hard centres. Bronwen chooses two sweets at random.

(c) What is the probability that neither of the sweets will have a hard centre?

14. Boris and John play three sets of tennis.

The probability of Boris winning the first set is 0.4. If Boris wins a set the probability of his winning the next set is 0.7. If John wins a set the probability of his winning the next set is 0.8.

(a) Copy and complete the tree diagram to show all the possible outcomes of winning.



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(b) Calculate the probability that John wins all *three* sets.

(c) Calculate the probability that Boris wins **at** *least* two sets.

15. The fast food chain Macduff's is running a competition.

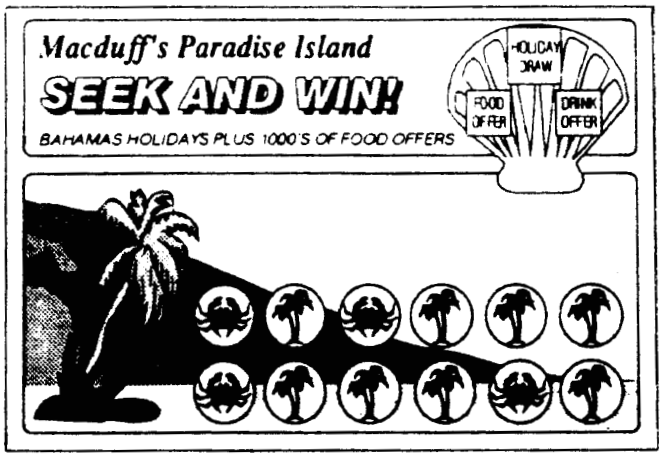
You obtain a card with 12 circles covered up.

You are allowed to scratch off up to three circles.

You win if 2 or more PALM TREES are revealed.

You lose if two or more CRABS are found.

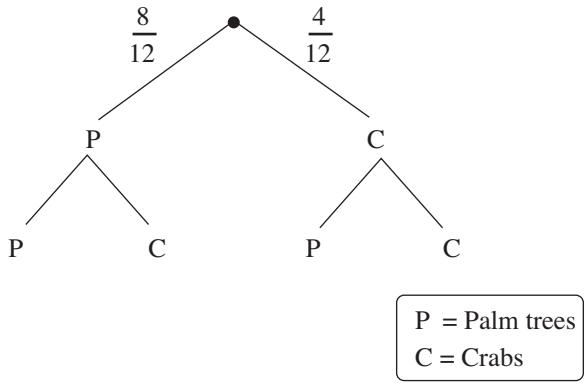
If you win, you scratch off one of the three squares to show your prize.



One of the cards is illustrated above with all the circles and squares revealed. The ratio of PALM TREES to CRABS is always 2:1 but their positions can change.

(a) The first circle you scratch off reveals a PALM TREE. What is the probability that the second circle reveals a PALM TREE?

(b) Copy and complete the tree diagram for all the winning combinations.

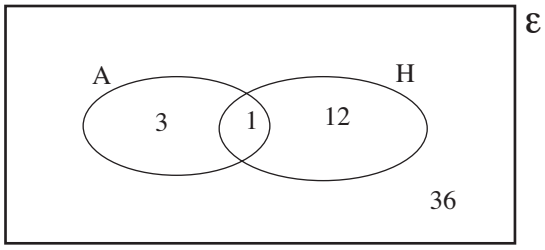


(c) List the possible winning combinations.

(d) What is the probability of winning?

D3.4 Using Venn Diagrams to Find Probabilities

Venn diagrams can be used to help find probabilities when events are not mutually exclusive.



This Venn diagram illustrates the situation for a pack of cards where the set A is the aces and the set H is the hearts. Note that the total of the entries is 52, and the complete set is denoted by ε.

From the diagram

$$p(\text{Ace of Hearts}) = \frac{1}{52}$$

$$p(\text{Ace or Heart}) = \frac{3 + 1 + 12}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Worked Example 1

In a class of 30 children there are 20 who can swim and 25 who can ride a bike. There is no child who cannot swim nor ride a bike.

Draw a Venn diagram and find the probabilities that a child selected at random:

(a) can swim and ride a bike,

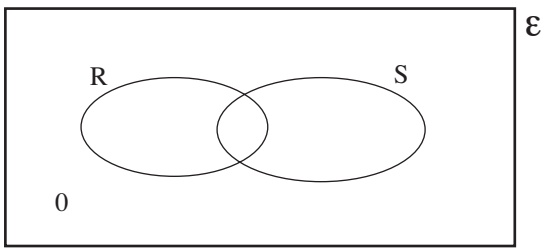
(b) cannot swim but can ride a bike,

(c) can swim but cannot ride a bike.

Solution

Let R represent the set of children that can ride a bike and S represent the set of children that can swim.

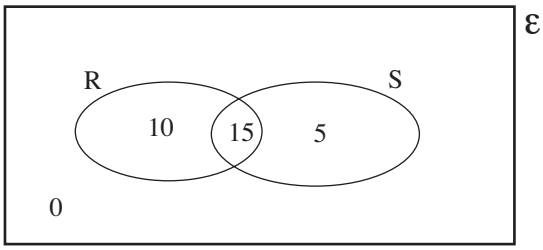
As each child belongs to R or S, a 0 can be placed as shown.



As $20 + 25 = 45$

and $45 - 30 = 15$

there must be 15 children in the overlap,
so the diagram can be completed as shown.



The probabilities can now be found:

$$p(\text{can swim and ride a bike}) = \frac{15}{30} = \frac{1}{2}$$

$$p(\text{cannot swim but can ride a bike}) = \frac{10}{30} = \frac{1}{3}$$

$$p(\text{can swim but cannot ride a bike}) = \frac{5}{30} = \frac{1}{6}.$$

Worked Example 2

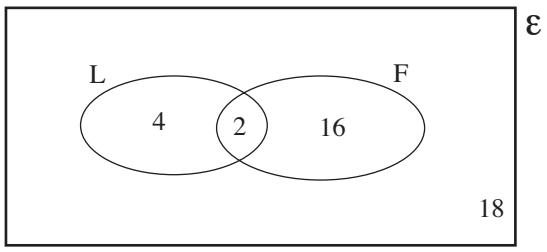
In a group of 40 students 6 are left-handed, 18 have size 7 feet and 2 are left-handed with size 7 feet. Draw a Venn diagram and find the probability that a student is :

(a) left-handed or has size 7 feet,

(b) not left-handed and does not have size 7 feet,

(c) not left-handed and has size 7 feet.

Solution



The number 2 is placed in the overlap

In F we have $18 - 2 = 16$,

In L we have $6 - 2 = 4$.

In the other space we have $40 - (4+2+16) = 18$.

Now the probabilities can be calculated.

$$p(\text{left-handed or size 7 feet}) = \frac{4+2+16}{40}$$
$$= \frac{22}{40}$$
$$= \frac{11}{20}$$

$$p(\text{not left-handed and does not have size 7 feet}) = \frac{18}{40}$$
$$= \frac{9}{20}$$

$$p(\text{not left-handed and has size 7 feet}) = \frac{16}{20}$$
$$= \frac{4}{5}$$



Exercises

1. The Venn diagrams represent the ways the events A and B can take place. For each case find

   (a) p(A and B) (b) p(A but not B)

   (c) p(A or B) (d) p(B but not A)

Diagram (i)

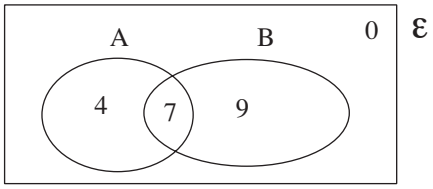


Diagram (ii)

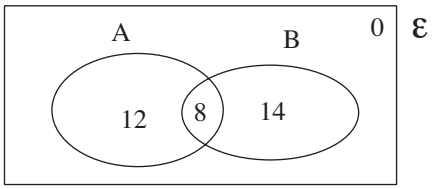


Diagram (iii)

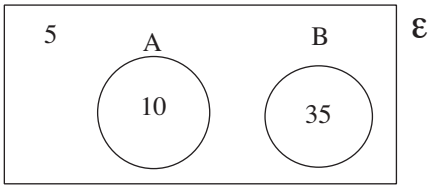
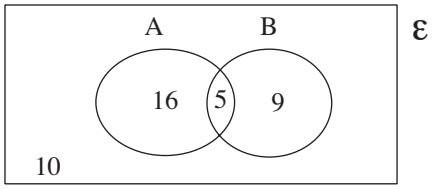


Diagram (iv)



2. When a card is selected at random from a full pack two possible events are:

   R : A red card is selected.

   Q : A queen is selected.

   Draw a Venn diagram and use it to find the probabilities that a card is:

   (a) a red queen,

   (b) red or a queen,

   (c) not red or a queen.

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3. In a survey 100 people were asked whether they watched snooker or cricket when it was on TV.

   20 watched neither, 80 watched snooker, 40 watched cricket.

   Draw a Venn diagram and find the probabilities that a person selected at random from the sample:

   (a) watched both cricket and snooker.

   (b) watched cricket but not snooker,

   (c) watched snooker but not cricket.

4. When a die is rolled the events defined below can be obtained.

   E : an even number,

   P : a prime number.

   Draw a Venn diagram and find the following probabilities.

   (a) p(E or P) (b) p(E and P)

   (c) p(E but not P) (d) P(neither E or P)

5. The sequence of lights at a set of traffic lights is:

Red	10 seconds
Red + Amber	2 seconds
Green	15 seconds
Amber	3 seconds

   Use a Venn diagram to find the following probabilities:

   (a) p(red light shows),

   (b) p(amber light shows),

   (c) p(red or amber lights show).

6. The following Venn diagrams show the ways in which the events A, B and C can take place. For each diagram find:

   (a) p(A and B and C) (b) p(A or B or C)

   (c) p(A or B) (d) p(A and B)

   (e) p(A or C) (f) p(A)

Diagram (i)

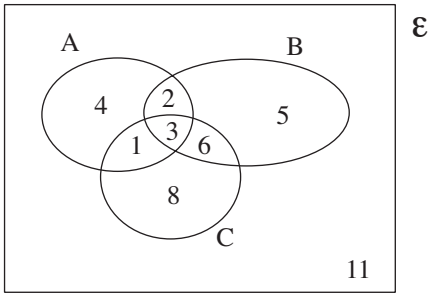


Diagram (ii)

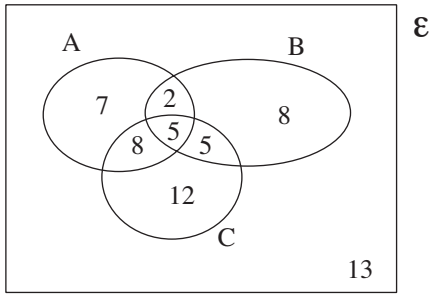


Diagram (iii)

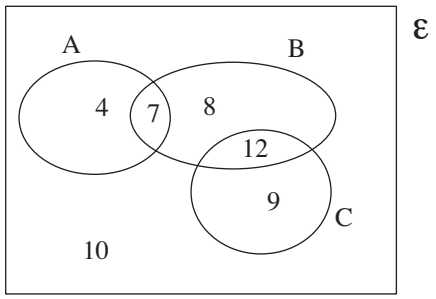
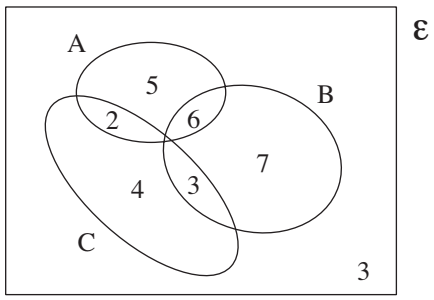


Diagram (iv)



7. When a person is selected at random, the probability that they are left handed is 0.08 and the probability that they are female is 0.52. If the probability that a female is left-handed is 0.042, find the probability that a person selected at random is female or left-handed.

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