STRAND E: Data Analysis

E3 *Measures of Central Tendency*

Text

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E3 Measures of Central Tendency

E3.1 Mean, Median, Mode and Range

In Units E1 and E2, you were looking at ways of collecting and representing data. In this unit, you will go one step further and find out how to calculate statistical quantities which summarise the important characteristics of the data.

The *mean*, *median* and *mode* are three different ways of describing the average.

- To find the *mean*, add up all the numbers and divide by the number of numbers.
- To find the *median*, place all the numbers in order and select the middle number.
- The *mode* is the number which appears most often.
- The *range* gives an idea of how the data are spread out and is the difference between the smallest and largest values.



Worked Example 1

Find

(a) the mean (b) the median (c) the mode (d) the range

of this set of data.

5, 6, 2, 4, 7, 8, 3, 5, 6, 6



Solution

(a) The mean is

$$\frac{5+6+2+4+7+8+3+5+6+6}{10}$$

$$= \frac{52}{10}$$

$$= 5.2$$

(b) To find the median, place all the numbers in order.

2, 3, 4, 5, 5, 6, 6, 6, 7, 8

As there are *two* middle numbers in this example, 5 and 6,

$$\text{median} = \frac{5+6}{2}$$

$$= \frac{11}{2}$$

$$= 5.5$$

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(c) From the list above it is easy to see that 6 appears more than any other number, so

$$\text{mode} = 6$$

(d) The range is the difference between the smallest and largest numbers, in this case 2 and 8. So the range is $8 - 2 = 6$.

Worked Example 2

Five people play golf and at one hole their scores are

$$3,\ 4,\ 4,\ 5,\ 7$$

For these scores, find

(a) the mean (b) the median (c) the mode (d) the range .

Solution

(a) The mean is

$$\frac{3+4+4+5+7}{5} = \frac{23}{5} = 4.6$$

(b) The numbers are already in order and the middle number is 4. So

$$\text{median} = 4$$

(c) The score 4 occurs most often, so,

$$\text{mode} = 4$$

(d) The range is the difference between the smallest and largest numbers, in this case 3 and 7, so

$$\text{range} = 7 - 3 = 4$$

Worked Example 3

In a survey of 10 households, the number of children was found to be

4,	1,	5,	4,	3,	7,	2,	3,	4,	1

(a) State the mode.

(b) Calculate

(i) the mean number of children per household

(ii) the median number of children per household.

(c) A researcher says: "The mode seems to be the best average to represent the data in this survey." Give ONE reason to support this statement.

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(d) Calculate the probability that a household chosen at random from those in the survey would have

(i) exactly 4 children

(ii) more than 4 children.

Solution

(a) Mode = 4 (as its frequency is highest)

(b) (i) Mean $= (4 + 1 + 5 + 4 + 3 + 7 + 2 + 3 + 4 + 1) \div 10$

$$= \frac{34}{10}$$

$$= 3.4$$

(ii) Median: first put the data in numerical order.

$$1,\quad 1,\quad 2,\quad 3,\quad \underbrace{3,\quad 4,}\quad 4,\quad 4,\quad 5,\quad 7$$

$$\text{median} = \frac{3+4}{2} = 3.5$$

(c) The mode gives the value that occurs most frequently.

(d) (i) $p(4 \text{ children}) = \frac{3}{10} = 0.3$ (4 occurs 3 times)

(ii) $p(\text{more than 4 children}) = \frac{2}{10} = 0.2$ (5 and 7)

Exercises

1. Find the mean, median, mode and range of each set of numbers below.

(a) 3, 4, 7, 3, 5, 2, 6, 10

(b) 8, 10, 12, 14, 7, 16, 5, 7, 9, 11

(c) 17, 18, 16, 17, 17, 14, 22, 15, 16, 17, 14, 12

(d) 108, 99, 112, 111, 108

(e) 64, 66, 65, 61, 67, 61, 57

(f) 21, 30, 22, 16, 24, 28, 16, 17

2. Twenty students were asked their shoe sizes. The results are given below.

8,	6,	7,	6,	5,	$4\frac{1}{2}$,	$7\frac{1}{2}$,	$6\frac{1}{2}$,	$8\frac{1}{2}$,	10
7,	5,	$5\frac{1}{2}$	8,	9,	7,	5,	6,	$8\frac{1}{2}$	6

For this data, find

(a) the mean (b) the median (c) the mode (d) the range.

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3. Eight people work in an office. They are paid hourly rates of

£12, £15, £15, £14, £13, £14, £13, £13

(a) Find

(i) the mean (ii) the median (iii) the mode.

(b) Which average would you use if you wanted to claim that the staff were:

(i) well paid (ii) badly paid?

(c) What is the range?

4. Two people work in a factory making parts for cars. The table shows how many complete parts they make in one week.

Worker	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*
Rachel	20	21	22	20	21
John	30	15	12	36	28

(a) Find the mean and range for Rachel and John.

(b) Who is more consistent?

(c) Who makes the most parts in a week?

5. A gardener buys 10 packets of seeds from two different companies. Each pack contains 20 seeds and he records the number of plants which grow from each pack.

Company A	20	5	20	20	20	6	20	20	20	8
Company B	17	18	15	16	18	18	17	15	17	18

(a) Find the mean, median and mode for each company's seeds.

(b) Which company does the mode suggest is best?

(c) Which company does the mean suggest is best?

(d) Find the range for each company's seeds.

6. Lionel takes four tests and scores the following marks.

65, 72, 58, 77

(a) What are his median and mean scores?

(b) If he scores 70 in his next test, does his mean score increase or decrease? Find his new mean score.

(c) Which has increased most, his mean score or his median score?

7. David keeps a record of the number of fish he catches over a number of fishing trips. His records are:

1, 0, 2, 0, 0, 0, 12, 0, 2, 0, 0, 1, 18, 0, 2, 0, 1.

(a) Why does he object to talking about the mode and median of the number of fish caught?

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(b) What are the mean and range of the data?

(c) David's friend, Evan also goes fishing. The mode of the number of fish he has caught is also 0 and his range is 15.
What is the largest number of fish that Evan has caught?

8. A petrol station owner records the number of cars which visit his premises on 10 days. The numbers are:

204, 310, 279, 314, 257, 302, 232, 261, 308, 217

(a) Find the mean number of cars per day.

(b) The owner hopes that the mean will increase if he includes the number of cars on the next day. If 252 cars use the petrol station on the next day, will the mean increase or decrease?

9. The students in a class state how many children there are in their family. The numbers they state are given below.

1, 2, 1, 3, 2, 1, 2, 4, 2, 2, 1, 3, 1, 2,
2, 2, 1, 1, 7, 3, 1, 2, 1, 2, 2, 1, 2, 3

(a) Find the mean, median and mode for this data.

(b) Which is the most sensible average to use in this case?

10. In a singing contest, the scores awarded by eight judges were:

5.9 6.7 6.8 6.5 6.7 8.2 6.1 6.3

(i) Using the eight scores, determine:
(a) the mean
(b) the median
(c) the mode

(ii) Only six scores are to be used. Which two scores may be omitted to leave the value of the **median** the same?

11. The table shows the maximum and minimum temperatures recorded in six cities one day last year.

City	*Maximum*	*Minimum*
Los Angeles	22°C	12°C
Boston	22°C	−3°C
Moscow	18°C	−9°C
Atlanta	27°C	8°C
Archangel	13°C	−15°C
Cairo	28°C	13°C

(a) Work out the range of temperature for Atlanta.

(b) Which city in the table had the lowest temperature?

(c) Work out the difference between the maximum temperature and the minimum temperature for Moscow.

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12. The weights, in grams, of seven sweet potatoes are

260, 225, 205, 240, 232, 205, 214

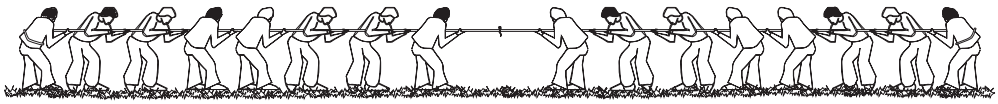
What is the median weight?

13. Here are the number of goals scored by a school football team in their matches this term.

3, 2, 0, 1, 2, 0, 3, 4, 3, 2

(a) Work out the mean number of goals.

(b) Work out the range of the number of goals scored.

14. 

(a) The weights, in kilograms, of the 8 members of *Hereward House* tug-of-war team at a school sports event are

75, 73, 77, 76, 84, 76, 77, 78.

Calculate the mean weight of the team.

(b) The 8 members of *Nelson House* tug of war team have a mean weight of 64 kilograms.

Which team do you think will win a tug-of-war between *Hereward House* and *Nelson House*? Give a reason for your answer.

15. Students in Year 8 are arranged in eleven classes. The class sizes are

23, 24, 24, 26, 27, 28, 30, 24, 29, 24, 27.

(a) What is the modal class size?

(b) Calculate the mean class size.

The range of the class sizes for Year 9 is 3.

(c) What does this tell you about the class sizes in Year 9 compared with those in Year 8?

16. A school has to select one student to take part in a General Knowledge Quiz.

Kelly and Rory took part in six trial quizzes. The following lists show their scores.

Kelly	28	24	21	27	24	26
Rory	33	19	16	32	34	18

Kelly had a mean score of 25 with a range of 7.

(a) Calculate Rory's mean score and range.

(b) Which student would you choose to represent the school? Explain the reason for your choice, referring to the mean scores and ranges.

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17. Eight judges each give a mark out of 6 in a gymnastics competition.

Nicole is given the following marks.

5.3, 5.7, 5.9, 5.4, 4.5, 5.7, 5.8, 5.7

The mean of these marks is 5.5, and the range is 1.4.

The rules say that the highest mark and the lowest mark are to be deleted.

5.3, 5.7, ~~5.9~~, 5.4, ~~4.5~~, 5.7, 5.8, 5.7

(a) (i) Find the mean of the six remaining marks.

(ii) Find the range of the six remaining marks.

(b) Do you think it is better to count all eight marks, or to count only the six remaining marks? Use the means and the ranges to explain your answer.

(c) The eight marks obtained by Diana in the same competition have a mean of 5.2 and a range of 0.6. Explain why none of her marks could be as high as 5.9.

E3.2 Finding the Mean from Tables and Tally Charts

Often data are collected into tables or tally charts. This section considers how to find the mean in such cases.



Worked Example 1

A football team keep records of the number of goals it scores per match during a season. The list is shown opposite.

Find the mean number of goals per match.

No. of Goals	*Frequency*
0	8
1	10
2	12
3	3
4	5
5	2



Solution

The previous table can be used, with a third column added.

No. of Goals	*Frequency*	*No. of Goals* × *Frequency*
0	8	0 × 8 = 0
1	10	1 × 10 = 10
2	12	2 × 12 = 24
3	3	3 × 3 = 9
4	5	4 × 5 = 20
5	2	5 × 2 = 10
TOTALS	**40**	**73**
	(Total matches)	(Total goals)

The mean can now be calculated.

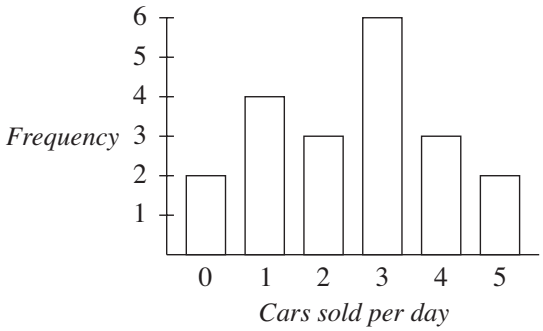
$$\text{Mean} = \frac{73}{40}$$

$$= 1.825$$

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Worked Example 2

The bar chart shows how many cars were sold by a salesman over a period of time.



Find the mean number of cars sold per day.

Solution

The data can be transferred to a table and a third column included as shown.

Cars sold daily	*Frequency*	*Cars sold* × *Frequency*
0	2	0 × 2 = 0
1	4	1 × 4 = 4
2	3	2 × 3 = 6
3	6	3 × 6 = 18
4	3	4 × 3 = 12
5	2	5 × 2 = 10
TOTALS	**20**	**50**
	(Total days)	(Total number of cars sold)

$$\text{Mean} = \frac{50}{20}$$

$$= 2.5 \text{ cars}$$

Worked Example 3

A police station kept records of the number of road traffic accidents in their area each day for 100 days. The figures below give the number of accidents per day.

1	4	3	5	5	2	5	4	3	2	0	3	1	2	2	3	0	5	2	1
3	3	2	6	2	1	6	1	2	2	3	2	2	2	2	5	4	4	2	3
3	1	4	1	7	3	3	0	2	5	4	3	3	4	3	4	5	3	5	2
4	4	6	5	2	4	5	5	3	2	0	3	3	4	5	2	3	3	4	4
1	3	5	1	1	2	2	5	6	6	4	6	5	8	2	5	3	3	5	4

Find the mean number of accidents per day.

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Solution

The first step is to draw out and complete a tally chart. The final column shown below can then be added and completed.

Number of Accidents	*Tally*	*Frequency*	*No. of Accidents* × *Frequency*
0	𝍸	4	0 × 4 = 0
1	𝍸 𝍸	10	1 × 10 = 10
2	𝍸 𝍸 𝍸 𝍸 ||	22	2 × 22 = 44
3	𝍸 𝍸 𝍸 𝍸 |||	23	3 × 23 = 69
4	𝍸 𝍸 𝍸 |	16	4 × 16 = 64
5	𝍸 𝍸 𝍸 ||	17	5 × 17 = 85
6	𝍸 |	6	6 × 6 = 36
7	|	1	7 × 1 = 7
8	|	1	8 × 1 = 8
	TOTALS	**100**	**323**

$$\text{Mean number of accidents per day} = \frac{323}{100} = 3.23$$

Worked Example 4

The marks obtained by 25 pupils on a test are shown below.

3	4	5	6	5
5	1	2	3	3
4	7	5	1	5
2	5	6	5	4
6	4	5	4	3

(a) Copy and complete the frequency table below to present the information given above.

Marks	*Frequency*
1	2
2	2
3	4
4	-
5	-
6	3
7	1

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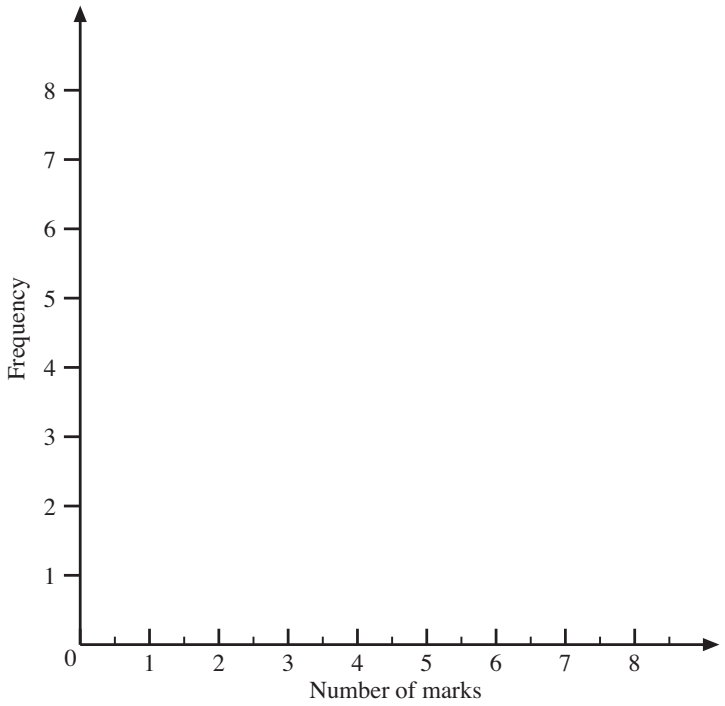
(b) Using the frequency distribution, state

(i) the modal mark

(ii) the median mark

(iii) the range.

(c) On graph paper, draw a histogram to illustrate the frequency distribution. Use axes as labelled below.



(d) A pupil is chosen at random from the group of pupils. What is the probability that the pupil's mark is greater than 5 ?

Solution

(a)

Marks	*Frequency*
1	2
2	2
3	4
4	5
5	8
6	3
7	1

(Check: total frequency = 2 + 2 + 4 + 5 + 8 + 3 + 1 = 25)

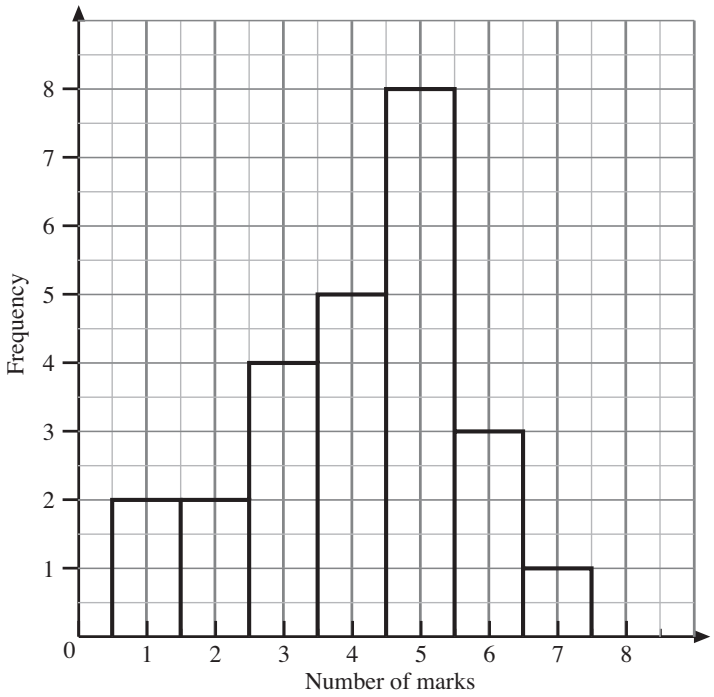
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(b) (i) Modal mark = 5 (with frequency 8)

(ii) Median mark = 4 (as we need the 13th number, when in order)

(iii) Range = 7 – 1 = 6

(c)



(d) $p(\text{mark greater than } 5) = \frac{3+1}{25} = \frac{4}{25} = 0.16$

Information

The study of statistics was begun by an English mathematician, John Graunt (1620–1674). He collected and studied the death records in various cities in Britain and, despite the fact that people die randomly, he was fascinated by the patterns he found.

Exercises

1. A survey of 100 households in an American town asked how many cars there were in each household The results are given below.

No. of cars	*Frequency*
0	5
1	70
2	21
3	3
4	1

Calculate the mean number of cars per household.

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2. The survey in question 1 also asked how many TV sets there were in each household. The results are given below.

No. of TV Sets	*Frequency*
0	2
1	30
2	52
3	8
4	5
5	3

Calculate the mean number of TV sets per household.

3. A manager keeps a record of the number of calls she makes each day on her mobile phone.

Number of calls per day	0	1	2	3	4	5	6	7	8
Frequency	3	4	7	8	12	10	14	3	1

Calculate the mean number of calls per day.

4. A cricket team keeps a record of the number of runs scored in each over.

No. of Runs	*Frequency*
0	3
1	2
2	1
3	6
4	5
5	4
6	2
7	1
8	1

Calculate the mean number of runs per over.

5. A class conduct an experiment in biology. They place a number of 1 m by 1 m square grids on the playing field and count the number of plants in each grid. The results obtained are given below.

6	3	2	1	3	2	1	3	0	1
0	3	2	1	1	4	0	1	2	0
1	1	2	2	2	4	3	1	1	1
2	3	3	1	2	2	2	1	7	1

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(a) Calculate the mean number of plants.

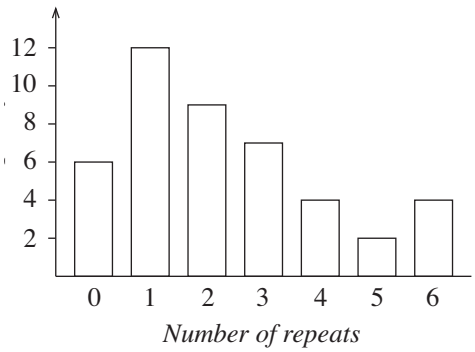
(b) How many times was the number of plants seen greater than the mean?

6. As part of a survey, the number of planes which were late arriving at Birmingham Airport each day was recorded. The results are listed below.

0	1	2	4	1	0	2	1	1	0
1	2	1	3	1	0	0	0	0	5
2	1	3	2	0	1	0	1	2	1
1	0	0	3	0	1	2	1	0	0

Construct a table and calculate the mean number of planes which were late each day.

7. Hannah drew this bar chart to show the number of repeated cards she got when she opened packets of football stickers.



Calculate the mean number of repeats per packet.

8. In a season a football team scored a total of 55 goals. The table below gives a summary of the number of goals per match.

Goals per Match	*Frequency*
0	4
1	6
2	
3	8
4	2
5	1

(a) In how many matches did they score 2 goals?

(b) Calculate the mean number of goals per match.

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9. A traffic warden is trying to work out the mean number of parking tickets he has issued per day. He produced the table below, but has accidentally rubbed out some of the numbers.

Tickets per day	*Frequency*	*No. of Tickets* × *Frequency*
0	1	●
1	●	1
2	10	●
3	7	●
4	●	20
5	2	●
6	●	●
TOTALS	**26**	**72**

Fill in the missing numbers and calculate the mean.

10. The number of children per family in a recent survey of 21 families is shown.

1	2	3	2	2	4	2	2
3	2	2	2	3	2	2	2
4	1	2	3	2			

(a) What is the range in the number of children per family?

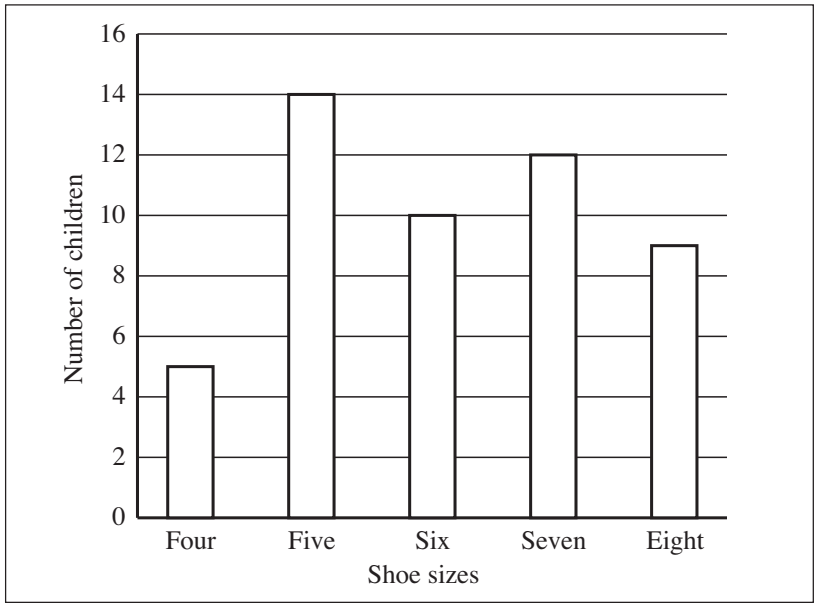
(b) Calculate the mean number of children per family. Show your working.

A similar survey was taken in 1980.

In 1980 the range in the number of children per family was 7 and the mean was 2.7.

(c) Describe **two** changes that have occurred in the number of children per family since 1980.

11. The bar chart below shows the shoe sizes of a group of 50 children.



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(a) How many children wear a size 7 shoe?

(b) How many children wear a shoe size smaller than size 7?

(c) Which shoe size is the **modal** size?

(d) What is the **median** shoe size?

(e) What is the probability that a child selected at random wears:

(i) a shoe size of 5?

(ii) a shoe size larger than 6?

(f) Which of these two averages, the mode and the median, would be of greater interest to the owner of a shoe shop who wishes to stock up on children's shoes? Give a reason for your answer.

E3.3 Calculations with the Mean

This section considers calculations concerned with the mean, which is usually taken to be the most important measure of the average of a set of data.



Worked Example 1

The mean of a sample of 6 numbers is 3.2. An extra value of 3.9 is included in the sample. What is the new mean?



Solution

$$\begin{aligned} \text{Total of original numbers} &= 6 \times 3.2 \\ &= 19.2 \\ \text{New total} &= 19.2 + 3.9 \\ &= 23.1 \end{aligned}$$

$$\text{New mean} = \frac{23.1}{7} = 3.3$$



Worked Example 2

The mean number of a set of 5 numbers is 12.7. What extra number must be added to bring the mean up to 13.1?



Solution

$$\begin{aligned} \text{Total of the original numbers} &= 5 \times 12.7 \\ &= 63.5 \\ \text{Total of the new numbers} &= 6 \times 13.1 \\ &= 78.6 \\ \text{Difference} &= 78.6 - 63.5 \\ &= 15.1 \end{aligned}$$

So the extra number is 15.1.

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Worked Example 3

Rohan's mean score in three cricket matches was 55 runs.

(i) How many runs did he score altogether?

After four matches his mean score was 61 runs.

(ii) How many runs did he score in the fourth match?



Solution

(i) $\text{Mean} = 55 = \dfrac{\text{total scored}}{3}$

so total scored $= 3 \times 55 = 165$

(ii) Total scored $= 4 \times 61 = 244$

Fourth match score $= 244 - 165$

$= 79$



Exercises

1. The mean height of a class of 28 students is 162 cm. A new student of height 149 cm joins the class. What is the mean height of the class now?

2. After 5 matches the mean number of goals scored by a football team per match is 1.8. If they score 3 goals in their 6th match, what is the mean after the 6th match?

3. The mean number of students ill at a school is 3.8 per day, for the first 20 school days of a term. On the 21st day 8 students are ill. What is the mean after 21 days?

4. The mean weight of 25 students in a class is 58 kg. The mean weight of a second class of 29 students is 62 kg. Find the mean weight of all the students.

5. A salesman sells a mean of 4.6 solar power systems per day for 5 days. How many must he sell on the sixth day to increase his mean to 5 sales per day?

6. Adrian's mean score for four test matches is 64. He wants to increase his mean to 68 after the fifth test. What does he need to score in the fifth test match?

7. The mean salary of the 8 people who work for a small company is £15000. When an extra worker is taken on this mean drops to £14000. How much does the new worker earn?

8. The mean of 6 numbers is 12.3. When an extra number is added, the mean changes to 11.9. What is the extra number?

9. When 5 is added to a set of 3 numbers the mean increases to 4.6. What was the mean of the original 3 numbers?

10. Three numbers have a mean of 64. When a fourth number is included the mean is doubled. What is the fourth number?

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11. Five numbers have a mean of 12. When one number is removed, the mean is 11. What is the value of the number removed?

12. 10 numbers have a mean of 7.5. The number 3 is removed. What is the new mean?

E3.4 Mean, Median and Mode for Grouped Data

The mean and median can be estimated from tables of *grouped* data.

The class interval which contains the most values is known as the *modal class*.

Worked Example 1

The table below gives data on the heights, in cm, of 51 children.

Class Interval	$140 \le h < 150$	$150 \le h < 160$	$160 \le h < 170$	$170 \le h < 180$
Frequency	6	16	21	8

(a) Estimate the mean height. (b) Find the median class.

(c) Find the modal class.

Solution

(a) To estimate the mean, the mid-point of each interval should be used.

Class Interval	*Mid-point*	*Frequency*	*Mid-point* × *Frequency*
$140 \le h < 150$	145	6	$145 \times 6 = 870$
$150 \le h < 160$	155	16	$155 \times 16 = 2480$
$160 \le h < 170$	165	21	$165 \times 21 = 3465$
$170 \le h < 180$	175	8	$175 \times 8 = 1400$
	Totals	**51**	**8215**

$$\text{Mean} = \frac{8215}{51}$$

$$= 161 \text{ (to the nearest cm)}$$

(b) The median is the 26th value. In this case it lies in the $160 \le h < 170$ class interval. Note that you can estimate the median height.

The 4th value in the interval is needed. It is estimated as

$$160 + \frac{4}{21} \times 10 = 162 \text{ (to the nearest cm)}$$

(c) The modal class is $160 \le h < 170$ as it contains the most values.

Also note that when we speak of someone by age, say 8, then the person could be any age from 8 years 0 days up to 8 years 364 days (365 in a leap year!). You will see how this is tackled in the following example.

Worked Example 2

The ages of students in a small primary school were recorded in the table below.

Age	5 – 6	7 – 8	9 – 10
Frequency	29	40	38

(a) Estimate the mean. (b) Estimate the median. (c) Find the modal class.

Solution

(a) To estimate the mean, we must use the mid-point of each interval; so, for example for '5 – 6', which really means

$$5 \leq \text{age} < 7$$

the mid-point is taken as 6.

Class Interval	*Mid-point*	*Frequency*	*Mid-point* × *Frequency*
5 – 6	6	29	$6 \times 29 = 174$
7 – 8	8	40	$8 \times 40 = 320$
9 – 10	10	38	$10 \times 38 = 380$
	Totals	**107**	**874**

$$\text{Mean} = \frac{874}{107}$$

$$= 8.2 \text{ (to 1 decimal place)}$$

(b) The median is given by the 54th value, which we have to estimate. There are 29 values in the first interval, so we need to estimate the 25th value in the second interval. As there are 40 values in the second interval, the median is estimated as being

$$\frac{25}{40}$$

of the way along the second interval. This has width $9 - 7 = 2$ years, so the median is estimated by

$$\frac{25}{40} \times 2 = 1.25$$

from the start of the interval. Therefore the median is estimated as

$$7 + 1.25 = 8.25 \text{ years}$$

(c) The modal class is the 7 – 8 age group.

Worked Example 1 uses what are called *continuous data*, since height can be of any value. (Other examples of continuous data are weight, temperature, area, volume and time.)

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The next example uses *discrete data*, that is, data which can take only a particular value, such as the integers $1, 2, 3, 4, \ldots$ in this case.

The calculations for mean and mode are not affected but estimation of the median requires replacing the *discrete* grouped data with an approximate *continuous* interval.



Worked Example 3

The number of days that students were missing from school due to sickness in one year was recorded.

Number of days off sick	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
Frequency	12	11	10	4	3

(a) Estimate the mean. (b) Find the median class. (c) Find the modal class.



Solution

(a) The estimate is made by assuming that all the values in a class interval are equal to the midpoint of the class interval.

Class Interval	*Mid-point*	*Frequency*	*Mid-point* × *Frequency*
1–5	3	12	$3 \times 12 = 36$
6–10	8	11	$8 \times 11 = 88$
11–15	13	10	$13 \times 10 = 130$
16–20	18	4	$18 \times 4 = 72$
21–25	23	3	$23 \times 3 = 69$
	Totals	**40**	**395**

$$\text{Mean} = \frac{395}{40}$$

$$= 9.875 \text{ days}$$

(b) As there are 40 students, we need to consider the mean of the 20th and 21st values. These both lie in the 6–10 class interval, which is really the 5.5–10.5 class interval, so this interval contains the median.

[You could also estimate the median as follows.

As there are 12 values in the first class interval, the median is found by considering the 8th and 9th values of the second interval.

As there are 11 values in the second interval, the median is estimated as being $\frac{8.5}{11}$ of the way along the second interval.

But the length of the second interval is $10.5 - 5.5 = 5$, so the median is estimated by

$$\frac{8.5}{11} \times 5 \approx 3.86$$

from the start of this interval. Therefore the median is estimated as

$$5.5 + 3.86 = 9.36 \ ]$$

(c) The modal class is 1–5, as this class contains the most entries.

Worked Example 4

The table shows the distribution of scores or 40 students on a Mathematics test.

Score	10 - 12	13 - 15	16 - 18	19 - 21	22 - 24
Frequency	4	6	13	9	8

(a) Estimate the mean score obtained on the test.

(b) Estimate the probability that a student selected at random would score at most 15 marks on the test.

(c) Using the scale of 1 cm to represent 1 unit on the frequency axis and 2 cm to represent 5 units on the scores axis, use graph paper to draw a frequency polygon to represent the distribution of scores shown in the table.

Solution

(a)

Score	Midpoint	Frequency
10 - 12	11	4
13 - 15	14	6
16 - 18	17	13
19 - 21	20	9
22 - 24	23	8

$$\text{Mean} = \left(11 \times 4 + 14 \times 6 + 17 \times 13 + 20 \times 9 + 23 \times 8\right) \div 40$$

$$= \left(44 + 84 + 221 + 180 + 184\right) \div 40$$

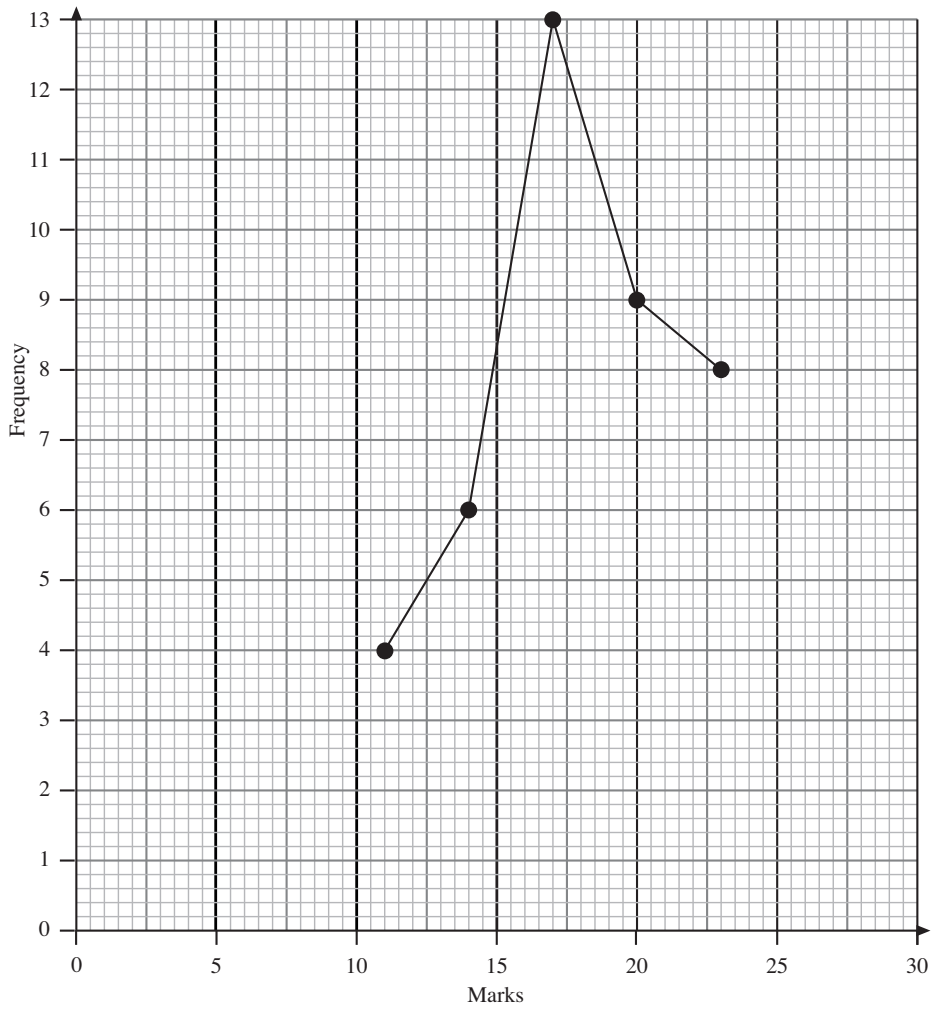
$$= \frac{713}{40}$$

$$= 17.825$$

(b) probability of score at most 15 $= \dfrac{11 + 14}{40} = \dfrac{25}{40} = = 0.625$

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(c)





Exercises

1. A salesman keeps a record of the number of shops he visits each day.

Shops visited	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
Frequency	3	8	24	60	21

(a) Estimate the mean number of shops visited.
(b) Estimate the median.
(c) What is the modal class?

2. The weights of a number of students were recorded in kg.

Weight (kg)	$30 \le w < 35$	$35 \le w < 40$	$40 \le w < 45$	$45 \le w < 50$	$50 \le w < 55$
Frequency	10	11	15	7	4

(a) Estimate the mean weight. (b) Estimate the median.
(c) What is the modal class?

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3. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (seconds)	$10 \le t < 15$	$15 \le t < 20$	$20 \le t < 25$	$25 \le t < 30$
Frequency	6	16	21	8

(a) Is the median in the modal class? (b) Estimate the mean.
(c) Estimate the median.
(d) Is the median greater or less than the mean?

4. The distances that students in a year group travelled to school is recorded.

Distance (km)	$0 \le d < 0.5$	$0.5 \le d < 1.0$	$1.0 \le d < 1.5$	$1.5 \le d < 2.0$
Frequency	30	22	19	8

(a) Does the modal class contain the median?
(b) Estimate the median and the mean.
(c) Which is the larger, the median or the mean?

5. The ages of the people at a youth camp are summarised in the table below.

Age (years)	6 – 8	9 – 11	12 – 14	15 – 17
Frequency	8	22	29	5

Estimate the mean age.

6. The lengths of a number of leaves collected for a project are recorded.

Length (cm)	2 – 5	6 – 10	11 – 15	16 – 25
Frequency	8	20	42	12

Estimate (a) the mean (b) the median length of a leaf.

7. The table shows how many nights people spend at a campsite.

Number of nights	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
Frequency	20	26	32	5	2

(a) Estimate the mean. (b) Estimate the median.
(c) What is the modal class?

8. (a) A teacher notes the number of correct answers given by a class on a multiple-choice test.

Correct answers	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Frequency	2	8	15	11	3

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(i) Estimate the mean. (ii) Estimate the median.

(iii) What is the modal class?

(b) Another class took the same test. Their results are given below.

Correct answers	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Frequency	3	14	20	2	1

(i) Estimate the mean. (ii) Estimate the median.

(iii) What is the modal class?

(c) How do the results for the two classes compare?

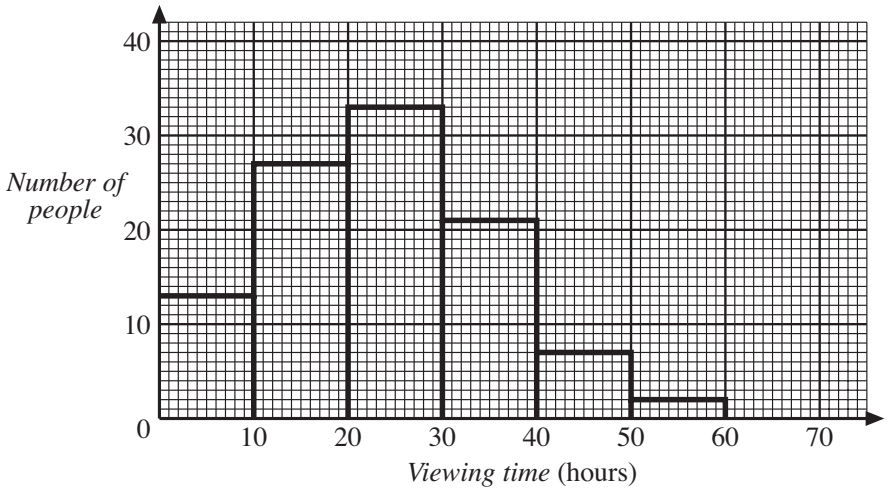
9. 29 students are asked how much money they were given at their last birthday. Their replies are shown in this frequency table.

'Birthday money' £	*Frequency* *f*
0 – £10.00	12
£10.01 – £20.00	9
£20.01 – £30.00	6
£30.01 – £40.00	2

(a) Which is the modal class?

(b) Calculate an estimate of the mean amount of money received per student.

10. The graph shows the number of hours a sample of people spent viewing television one week during the summer in London.



(a) Copy and complete the following frequency table for this sample.

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Viewing time (h hours)	*Number of people*
$0 \le h < 10$	13
$10 \le h < 20$	27
$20 \le h < 30$	33
$30 \le h < 40$	
$40 \le h < 50$	
$50 \le h < 60$	

(b) Another survey is carried out during the winter. State **one** difference you would expect to see in the data.

(c) Use the mid-points of the class intervals to calculate the mean viewing time for these people. You may find it helpful to use the table below.

Viewing time (h hours)	*Mid-point*	*Frequency*	*Mid-point* × *Frequency*
$0 \le h < 10$	5	13	65
$10 \le h < 20$	15	27	405
$20 \le h < 30$	25	33	825
$30 \le h < 40$	35		
$40 \le h < 50$	45		
$50 \le h < 60$	55		

11. In an experiment, 50 people were asked to estimate the length of a rod to the nearest centimetre. The results were recorded.

Length (cm)	20	21	22	23	24	25	26	27	28	29
Frequency	0	4	6	7	9	10	7	5	2	0

(a) Find the value of the median.

(b) Calculate the mean length.

(c) In a second experiment another 50 people were asked to estimate the length of the same rod. The most common estimate was 23 cm. The range of the estimates was 13 cm.

Make two comparisons between the results of the two experiments

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12. The following list shows the maximum daily temperature, in °F, throughout the month of April in London.

56.1	49.4	63.7	56.7	55.3	53.5	52.4	57.6	59.8	52.1
45.8	55.1	42.6	61.0	61.9	60.2	57.1	48.9	63.2	68.4
55.5	65.2	47.3	59.1	53.6	52.3	46.9	51.3	56.7	64.3

(a) Copy and complete the grouped frequency table below.

Temperature, T		*Frequency*
$40 < T \leq 50$		
$50 < T \leq 54$		
$54 < T \leq 58$		
$58 < T \leq 62$		
$62 < T \leq 70$		

(b) Use the table of values in part (a) to calculate an estimate of the mean of this distribution. You must show your working clearly.

(c) Draw a histogram to represent your distribution in part (a).

13. The table below shows the distribution of marks for 100 students on a science test.

Marks *(%)*	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90
Frequency	11	6	19	10	10	19	16	9

(a) (i) State the median class.

(ii) Obtain an estimate for the mean mark on the test.

(iii) Calculate the probability that a student chosen at random scored between 31 and 60, both scores inclusive.

(b) Using graph paper and a scale of 1 cm for 10 marks on the *x*-axis and 1 cm for 2 students on the *y*-axis, draw a frequency polygon to represent the information in the table.

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