F4 *Solving Quadratic Equations* Data Sheets

Data Sheets

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Data Sheet F4.1 *Solving Quadratic Equations by Factorisation*

A quadratic equation is of the form $\boxed{ax^2 + bx + c = 0}$ and can be solved if it factorises.

Example 1 Solve $x^2 + 6x + 5 = 0$

Solution Factorising $(x + \square)(x + \square) = 0$

so either $x + \square = 0$ or $x + \square = 0$, giving

solution $x = \square$ or $x = \square$

Example 2 Solve $x^2 - 12x = 0$

Solution Factorising $x(x - \square) = 0$

so either $x = 0$ or $x = \square$

Example 3 Solve $4x^2 - 81 = 0$ (difference of two squares)

Solution Factorising $(2x - \square)(2x + \square) = 0$

so either $x = \dfrac{\square}{2}$ or $x = \dfrac{\square}{2}$

Example 4 Solve $x^2 - 4x + 4 = 0$ (perfect square)

Solution Factorising $(x - \square)(x - \square) = 0$

so $x = \square$

This is a repeated solution so this quadratic has just one solution.

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Data Sheet F4.2a

Solving Quadratic Equations Using the Formula 1

There is a formula to solve the quadratic equation

$$ax^2 + bx + c = 0$$

The solution is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Using the formula, solve the quadratic equation

$$x^2 + 6x - 8 = 0$$

giving the solution correct to 2 decimal places.

Solution Here $a =$ ☐ , $b =$ ☐ , $c =$ ☐ , so

$$x = \frac{-\square \pm \sqrt{\square}}{2 \times \square}$$

$$= \frac{-\square \pm \sqrt{\square}}{\square}$$

$$= \frac{-\square + \sqrt{\square}}{\square} \text{ or } \frac{-\square - \sqrt{\square}}{\square}$$

To 2 d.p. $x =$ ☐ or $x =$ ☐

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Data Sheet F4.2b

Solving Quadratic Equations Using the Formula 2

There is a formula to solve the quadratic equation

$$ax^2 + bx + c = 0$$

The solution is given by

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 2 Using the formula, solve the quadratic equation

$$4x^2 - 12x + 9 = 0$$

Solution Here $a =$ ☐, $b =$ ☐, $c =$ ☐ and

$$x = \frac{-\square \pm \sqrt{\square}}{2 \times \square}$$

$$= \frac{-\square \pm \sqrt{\square}}{\square}$$

$$= \frac{\square}{\square}$$

There is only one distinct root as $\boxed{b^2 = 4ac}$

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Data Sheet F4.2c

Solving Quadratic Equations Using the Formula 3

There is a formula to solve the quadratic equation

$$ax^2 + bx + c = 0$$

The solution is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3 Using the formula, solve the quadratic equation

$$x^2 + x + 5 = 0$$

Solution Here $a =$ ☐, $b =$ ☐, $c =$ ☐ and

$$x = \frac{-\square \pm \sqrt{\square}}{2 \times \square}$$

$$= \frac{-\square \pm \sqrt{\square}}{\square}$$

This has no solution as ☐ < 0.

This is the case when $b^2 < 4ac$

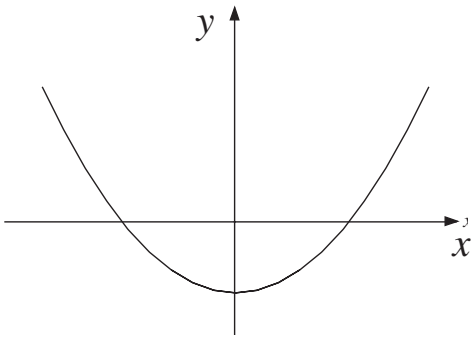
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Data Sheet F4.3

Summary of Number of Solutions

A quadratic equation can have 2, 1 or 0 solutions.

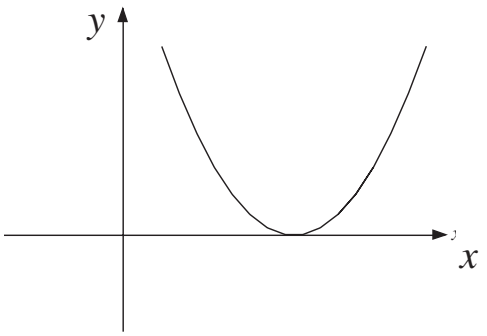
The graphs below illustrate these graphically and show how the number of solutions depends on the sign of $\left(b^2 - 4ac\right)$ which is part of the quadratic formula.



$b^2 - 4ac > 0$

No. of solutions = ☐

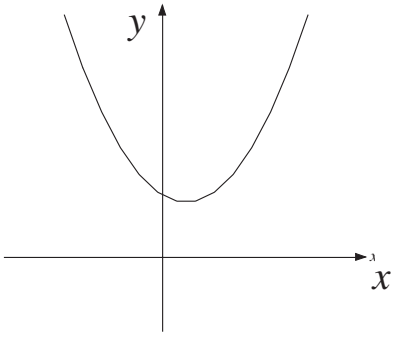
(See Example 1)



$b^2 = 4ac$

No. of solutions = ☐

(See Example 2)



$b^2 < 4ac$

No. of solutions = ☐

(See Example 3)

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Data Sheet F4.4

Completing the Square

The formula for solving a quadratic equation comes from completing the square.

Example Express $3x^2 + 2x + 1$ in the form $a(x + p)^2 + q$ when a, p and q are real numbers.

Solution

$$3x^2 + 2x + 1 = a(x + p)^2 + q$$

$$= a(x^2 + 2xp + p^2) + q$$

$$= \square\, x^2 + \square\, x + \square$$

So, comparing coefficients,

$[x^2]$ $3 = \square \Rightarrow a = \square$

$[x]$ $2 = \square \Rightarrow p = \dfrac{\square}{\square}$

$[\text{constant}]$ $1 = \square \Rightarrow q = \dfrac{\square}{\square}$

Thus $3x^2 + 2x + 1 = \square \left(x + \dfrac{\square}{\square}\right)^2 + \dfrac{\square}{\square}$

Extension

What is the minimum value of $3x^2 + 2x + 1$?

This is $\dfrac{\square}{\square}$ and it occurs when $x = -\dfrac{\square}{\square}$

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