F5 *Solving Equations* Introduction

Learning objectives

This unit extends students' algebraic skills by solving more complicated equations. After completing Unit F5 you should

- ✻ • be able to manipulate algebraic fractions
- ✻ • be able to solve quadratic equations that are derived from algebraic equations with fractions
- ✻ • understand how to solve simultaneous equations where one is linear and one is quadratic.

Introduction

This is the fifth unit in this series of 6 units on *Algebra*. Here we build on the work and techniques already introduced in Units F1 to F4.

Key points and principles

- Algebraic fractions are dealt with in a similar way to fractions.
- In solving simultaneous equations when one is linear and one is quadratic, substitute the linear equation with the quadratic, simplify the equation and then solve.

Facts to remember

- $\dfrac{1}{x+a}+\dfrac{1}{x-a}=\dfrac{2x}{x^2-a^2}$
- When solving equations of the form

  $$\frac{x+a}{x+b}+\frac{x+c}{x+d}=e$$

  you multiply both sides by $(x+b)(x+d)$
- The solution of simultaneous equations, one linear and one quadratic, can have 0, 1 or 2 solutions: in this unit you will meet only simultaneous equations with 2 solutions.

Glossary of terms

Algebraic fractions These are of the form

$$\frac{ax+b}{cx+d}$$

when a, b, c, and d are constants.

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F5 *Solving Equations* Introduction

For example, $\dfrac{x+3}{3x-1}$, $\dfrac{2y+4}{3-2y}$, $\dfrac{3}{x+2}$

(iii) ***completing the square***, where we write

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

- Note that if $(x-a)(x-b) = 0$, then either $x - a = 0$ or $x - b = 0$, giving $x = a$ or $x = b$.

- Quadratic equations have 2, 1 or 0 real roots according to the value of "$b^2 - 4ac$":

  (i) If $b^2 > 4ac$, there are 2 real distinct roots

  (ii) If $b^2 = 4ac$, there is 1 (repeated) real root

  (iii) If $b^2 < 4ac$, there are no real roots.

Facts to remember

- If $(x-a)(x-b) = 0$ then $x = a$ or $x = b$

- The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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F5 *Solving Equations* Introduction

- The number of real roots of a quadratic is

$$2 \text{ if } b^2 > 4ac$$

$$1 \text{ if } b^2 = 4ac$$

$$0 \text{ if } b^2 < 4ac$$

Glossary of Terms

Quadratic factorisation This is when you can write a quadratic formula in the form

$$ax^2 + bx + c = a(x - p)(x - q)$$

Formula for solving quadratic equation The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square This is when we write the quadratic in the form

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

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