G1 *Coordinates* Introduction

Learning objectives

This is the first of four units based on analytic geometry, which brings algebra and geometry together. After completing Unit G1 you should

- be able to identify and illustrate points in two dimensions with positive coordinates
- be able to identify and illustrate points in two dimensions
- be confident in plotting straight lines
- be confident in plotting curves
- ✻ be able to identify and illustrate points in three dimensions
- ✻ be able to find the mid-point of a line segment.

Introduction

The inspiration behind the development of graphs to represent functions is commonly credited to *René Descartes* (1596-1650) but the much earlier work of the Frenchman, *Nicole d'Oresme* (c. 1323–1382) represented the real starting point. d'Oresme was essentially a clergyman, being appointed canon and later dean of Rouen, and in 1370, appointed chaplain to King Charles V. His contribution to this area of mathematics was to find the logical equivalence between tabulating and graphing values. He proposed the use of a graph for plotting a variable magnitude when one value depends on another.



Portrait of René Descartes (after Frans Hals, 1648)

It is possible that the work of *Descartes* was influenced by *d'Oresme's* work, which was reprinted several times during the century following its first publication; but, despite this possible influence, the concept of the use of coordinate axes for representing functions is clearly a mathematical landmark.

As the inventor of the Cartesian coordinate system, Descartes founded analytic geometry, the bridge between algebra and geometry, crucial to the discovery of infinitesimal calculus and analysis. He is best known for the philosophical statement

"*Cogito ergo sum*"

(French: "Je pense, donc je suis"; English: "I think, therefore I am"; or "I am thinking, therefore I exist".)

Nowadays, we take coordinate axes very much for granted and a function and the graph to represent it, are almost regarded as being totally equivalent. Indeed, for the functions dealt with in this unit, there is little difference, but it should be noted that not all functions can be adequately represented by a graph.



Portrait of Nicole Oresme: Miniature of Nicole Oresme's Traité de l'espere, Bibliothèque Nationale, Paris, France

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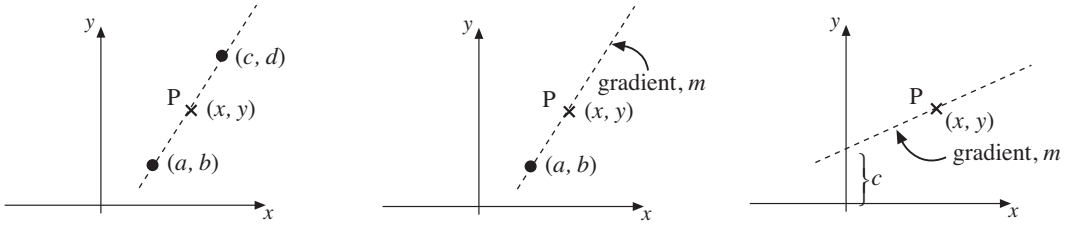
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There should also be a clear distinction made between graphs of, for example, straight lines, circles, etc. which are precise, and the graphs of lines or curves which are used to represent data. The former are exact, and, for example, a straight line is consequently determined by either

two points

or

one point and *a gradient*.



This results in the equation

$$\frac{y-b}{x-a} = \frac{d-b}{c-a}$$

or

$$\frac{y-b}{x-a} = m$$

The form typically used in this unit, which reflects the *CXC* requirements, is

$$y = mx + c$$

– but please note that all these are equivalent.

The last of these equations is of most use when attempting to represent data points by a straight line relationship (i.e. modelling), when it is easy to read off values for both m and c. So, in this case, you are turning experimental data into an algebraic relationship. This link between formal algebra and visual geometry lies behind the use of graphs in modelling.

One other point that is worth stressing is the use of a graphical approach to solving equations. Whilst this is of some benefit in finding *approximate* solutions to cubic equations, or other more complicated functions, there is little point in using a graphical approach to, for example, solve simultaneous linear equations – except that this is what most candidates will be expected to do in their exams!

Key points and principles

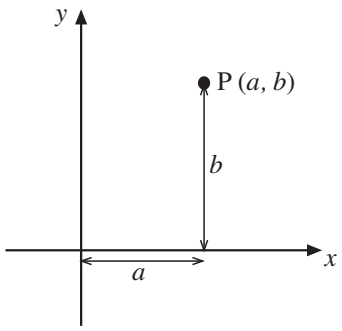
- Any point in 2 dimensions is uniquely defined by its coordinates (x, y)
- Any two distinct points are sufficient to define a straight line.
- Any point in 3 dimensions is uniquely defined by its coordinates (x, y, z).

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Facts to remember

- The point with coordinates (a, b) means that $x = a,\ y = b$.

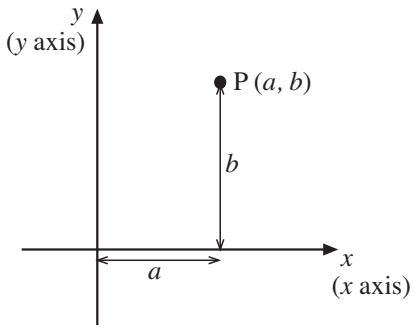


- The mid-point of the line segment joining the points (a, b) and (c, d) is given by $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

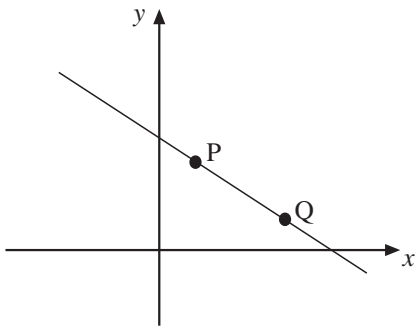
Glossary of terms

Coordinates (x, y) means that the point P with coordinates (a, b) is such that $x = a,\ y = b$.

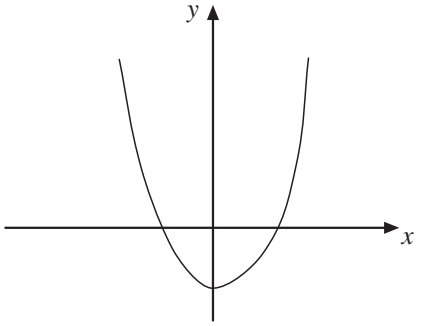
Coordinate axes these are shown on the diagram opposite.



Straight line is defined by any two points on the line P and Q in the diagram opposite.



Curves these are lines that are not straight (see diagram opposite).



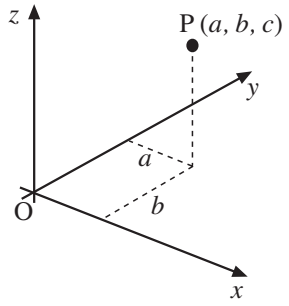
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Three-dimensional coordinates mean that a point P (a, b, c) is such that

$$x = a,\ y = b,\ z = c$$

(see diagram opposite).



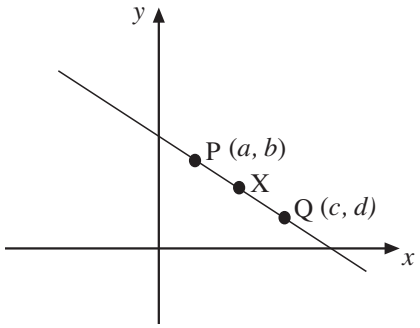
Line segment this is any part of a straight line between two points.

Mid-point of line segment This is the point X in the line segment PQ, such that

$$\text{length PX} = \text{length XQ}.$$

The coordinates of X are

$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$



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