STRAND G: Relations, Functions and Graphs

Unit G1 *Coordinates*

Text

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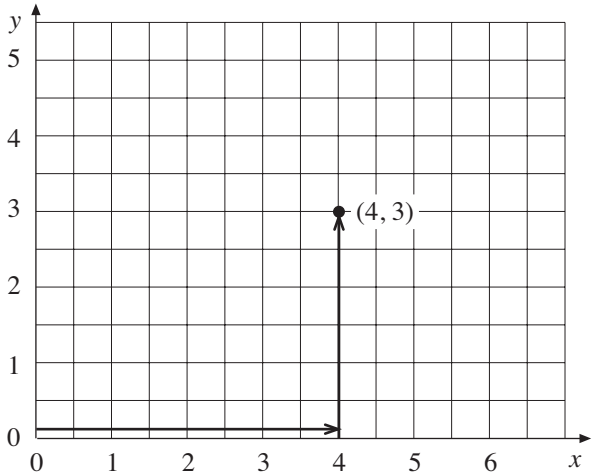
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G1 Coordinates

G1.1 Positive Coordinates

Coordinates are pairs of numbers that uniquely describe a position on a rectangular grid. The system was invented by the 17th century French mathematician, René Descartes. These numbers are sometimes referred to as *Cartesian coordinates*.

The first number refers to the horizontal (x-axis) and the second the vertical (y-axis). The coordinates (4, 3) describe a point that is 4 units across and 3 units up on a grid from the origin (0, 0).



Worked Example 1

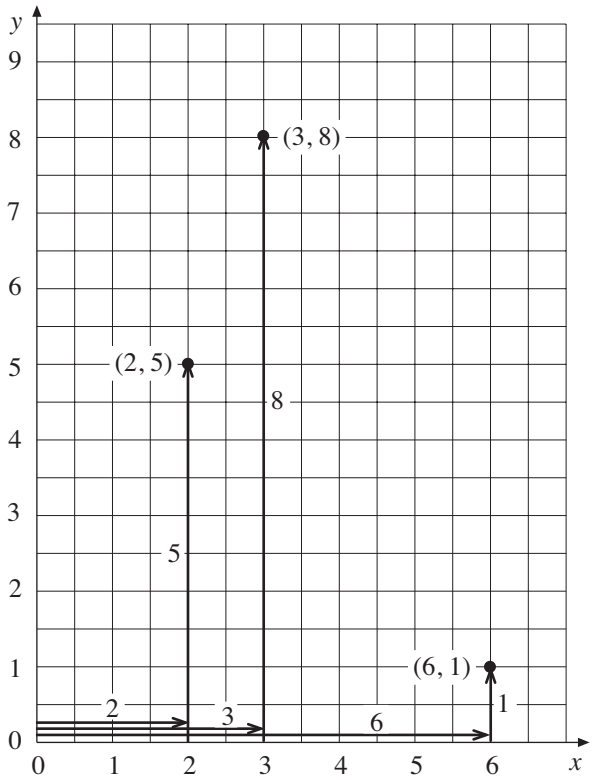
Plot the points with coordinates
(3, 8), (6, 1) and (2, 5)

Solution

For (3, 8) move 3 across and 8 up.

For (6, 1) move 6 across and 1 up.

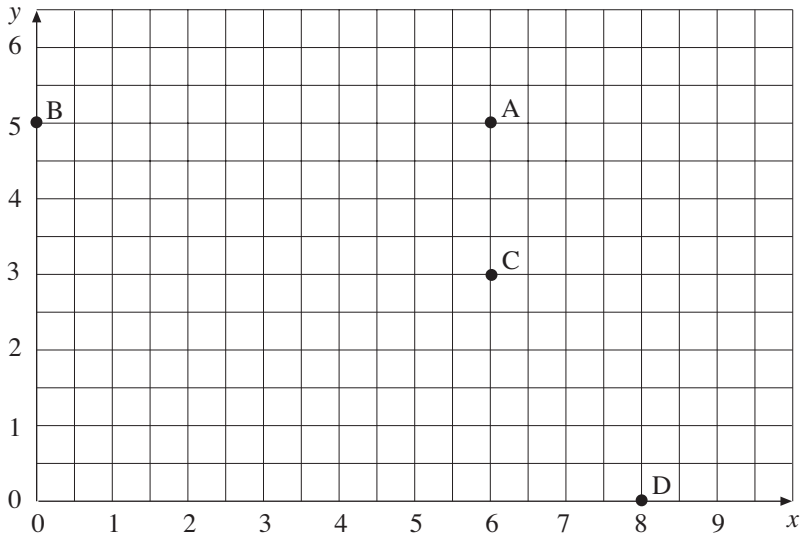
For (2, 5) move 2 across and 5 up.



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Worked Example 2

Write down the coordinates of each point in the diagram below.



Solution

A is 6 across and 5 up, so the coordinates are $(6, 5)$.

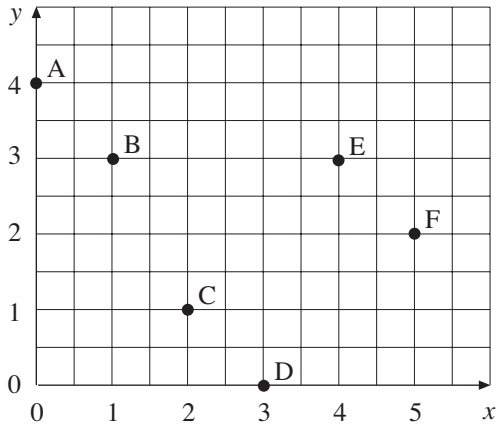
B has no movement across and is straight up 5, so the coordinates are $(0, 5)$.

C is 6 across and 3 up, so the coordinates are $(6, 3)$.

D is 8 across and no movement up, so the coordinates are $(8, 0)$.

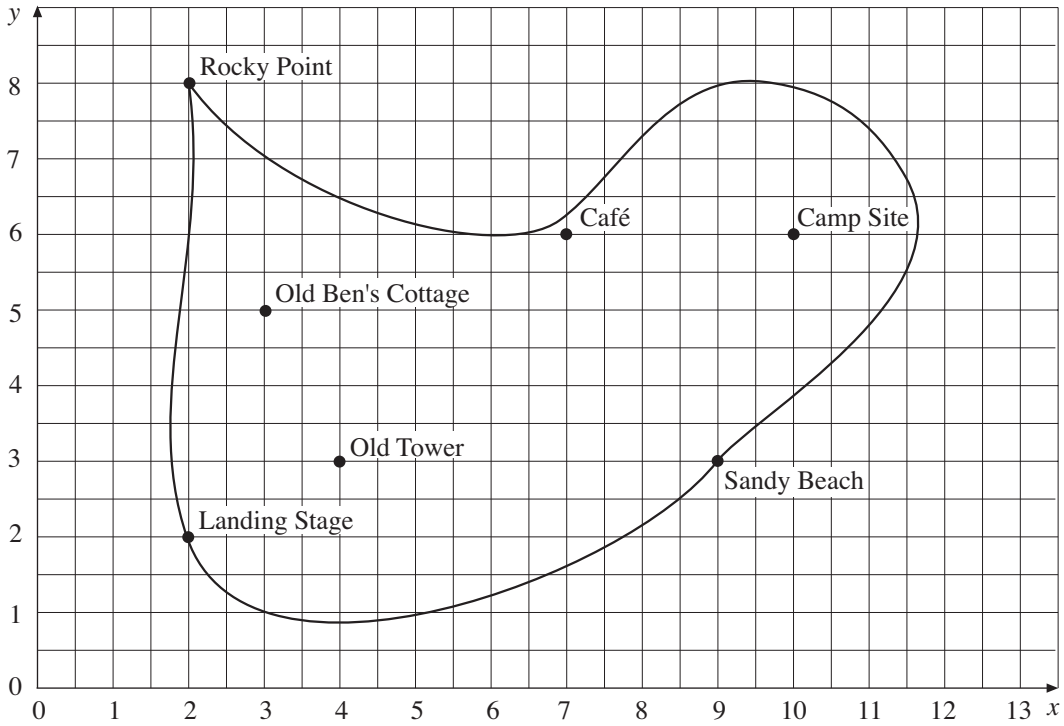
Exercises

1. Write down the coordinates of each point on the diagram below.



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2. The map of an island has been drawn on a grid.



Write down the coordinates of each place marked on the map.

3. On a grid, join the points with the following coordinates and write down the name of the shape you draw.

(a) (4, 2) (8, 2) (8, 5) (4, 5)

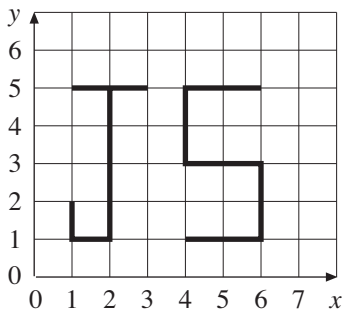
(b) (2, 1) (6, 1) (4, 6)

(c) (1, 4) (3, 7) (5, 4) (3, 1)

(d) (4, 0) (3, 2) (5, 4) (7, 2) (6, 0)

(e) (1, 1) (0, 3) (1, 5) (3, 5) (4, 3) (3, 1)

4. Javina writes her initials on a grid.

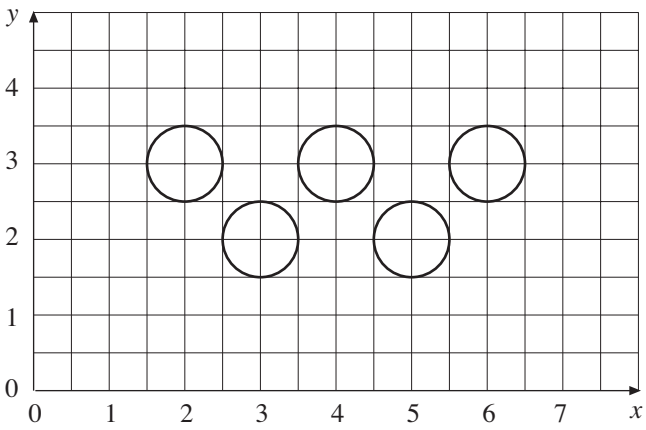


(a) Write down the coordinates of the corners of each letter.

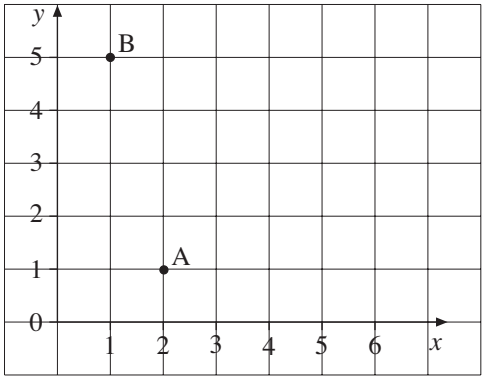
(b) Write your initials in the same way and write down the coordinates of your initials.

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5. The pattern below is made up of 5 circles. Write down the coordinates of the centre of each circle.



6.



(a) On the co-ordinate grid above plot the following points

P (3,4), Q (0,2) R (4,0)

(b) Write down the co-ordinates of the points

(i) A,

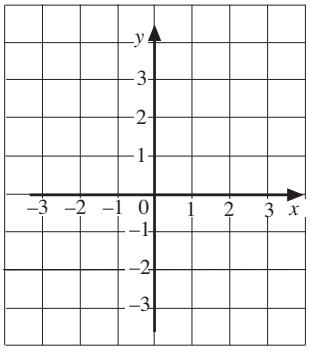
(ii) B.

G1.2 Coordinates

The coordinates of a point are written as a pair of numbers, (x, y), which describe where the point is on a set of axes.

The x-axis is always horizontal (i.e. *across* the page) and the y-axis always vertical (i.e. *up* the page).

The x-coordinate is always given first and the y-coordinate second.



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Worked Example 1

On a grid, plot the point A which has coordinates $(-2, 4)$, the point B with coordinates $(3, -2)$ and the point C with coordinates $(-4, -3)$.

Solution

For A, begin at $(0, 0)$, where the two axes cross.

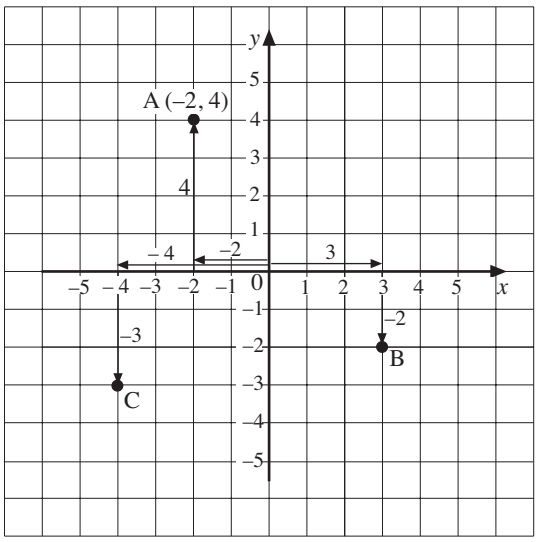
Move -2 in the x direction.

Move 4 in the y direction.

Points B and C are plotted in a similar way.

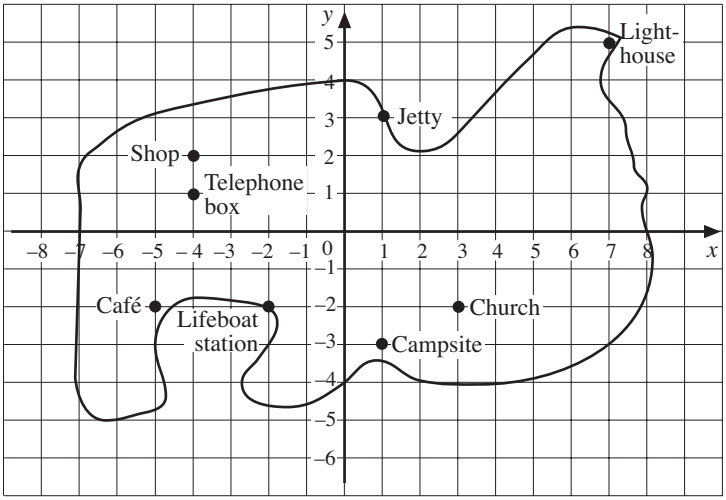
For B, move 3 in the x direction and -2 in the y direction.

For C, move -4 in the x direction and -3 in the y direction.



Worked Example 2

Write down the coordinates of each place on the map of the island.



Solution

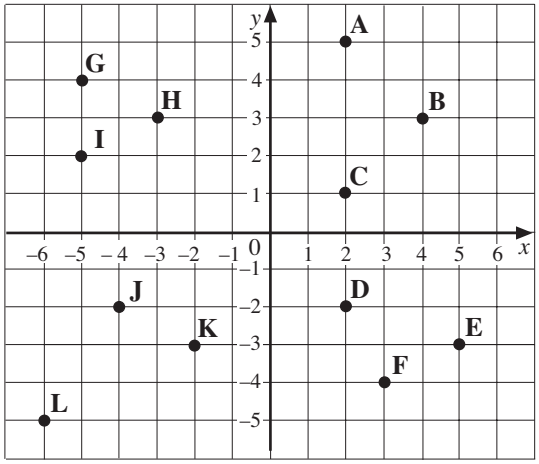
Lighthouse	(7, 5)	All coordinates positive
Jetty	(1, 3)	
Church	(3, –2)	Negative y-coordinates
Camp Site	(1, –3)	
Shop	(–4, 2)	Negative x-coordinates
Telephone Box	(–4, 1)	

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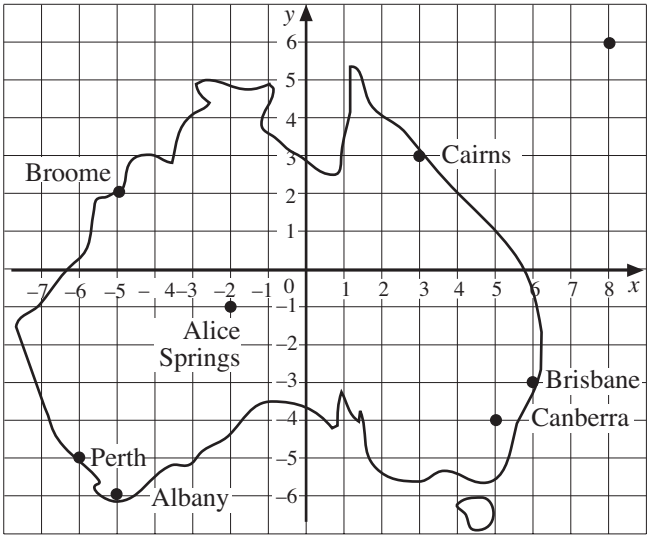
Café	(– 5, – 2)	All coordinates negative
Lifeboat Station	(– 2, – 2)	

Exercises

1. Write down the coordinates of each point marked on the grid below.



2. The map shows some Australian towns and cities.



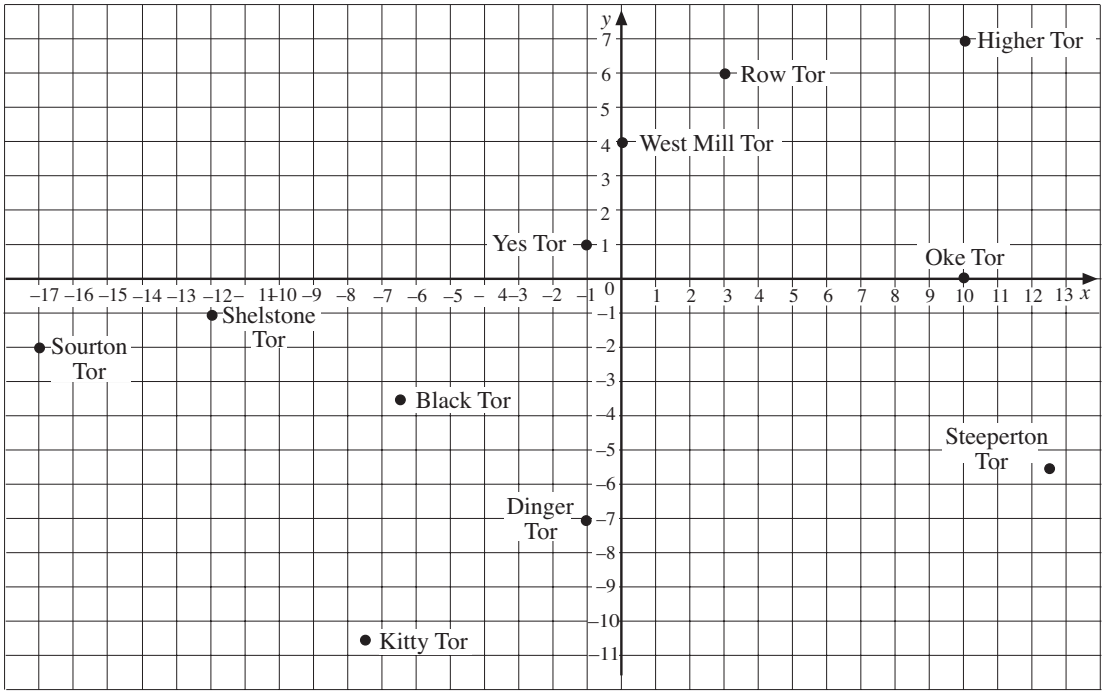
(a) Write down the coordinates of Canberra, Brisbane and Perth.

(b) A plane flies from the place with coordinates (–5, –6) and lands at the place with coordinates (–2, –1). From where does the plane take off and where does it land?

(c) A ship has coordinates (–5, 2) at the start of a voyage and coordinates (–6, –5) at the end. Where does it start and where does it finish?

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3. The positions of some of the tors (rocky outcrops) on Dartmoor in the south west of England are marked on the grid below.



(a) Write down the coordinates of the following tors.

West Mill Tor

Steeperton Tor

Shelstone Tor

Black Tor

Dinger Tor

(b) The highest tor marked on the grid is Yes Tor. Write down the coordinates of this tor.

(c) A boy and his dog walk from Oke Tor to Kitty Tor. Write down the coordinates of the point where they start and the point where they finish.

(d) Sourton Tor is the tor that is the farthest west on this grid. What are the coordinates of this tor?

(e) Higher Tor is the tor that is farthest north. What are the coordinates of this tor?

4. Draw a set of axes with x-values from –5 to 5 and y-values from –3 to 9.

(a) Join together the points with coordinates $(5, 0), (0, 9)$ and $(–5, 0)$. What shape do you get?

(b) On the same diagram, join together the points $(5, 6)$, $(–5, 6)$ and $(0, –3)$. What shape have you obtained?

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5. Draw a set of axes with x-values from -4 to 4 and y-values from -4 to 3.

   Join each set of points below in the order listed.

   (a) $(2, 0)$, $(1, 1)$, $(1, 2)$, $(2, 3)$, $(3, 3)$, $(4, 2)$, $(4, 1)$, $(3, 0)$.

   (b) $(0, 1)$, $(1, -1)$, $(-1, -1)$, $(0, 1)$.

   (c) $(-2, 0)$, $(-1, 1)$, $(-1, 2)$, $(-2, 3)$, $(-3, 3)$, $(-4, 2)$, $(-4, 1)$, $(-3, 0)$.

   (d) $(3, -1)$, $(2, -3)$, $(0, -4)$, $(-2, -3)$, $(-3, -1)$, $(-1, -2)$, $(1, -2)$, $(3, -1)$.

6. (a) Draw a set of axes with x-values from -4 to 4 and y-values from -5 to 4.

   (b) Plot the following points and join them in the order listed.

   $(3, -5)$, $(2, -5)$, $(-4, -2)$, $(-2, -3)$, $(0, -2)$, $(0, 0)$, $(3, 2)$, $(3, 3)$, $(4, 3)$, $(4, 2)$, $(3, 2)$, $(3, 0)$, $(-2, 2)$.

7. Three corners of a square have coordinates $(4, 2)$, $(-2, 2)$ and $(4, -4)$.

   (a) Draw a set of axes with x-values from -2 to 4 and y-values from -4 to 2. Plot the three points and draw the square.

   (b) Write down the coordinates of the centre of the square.

8. Two corners of a rectangle have coordinates $(-3, -1)$ and $(-3, 3)$. The centre of the rectangle has coordinates $(1, 1)$.

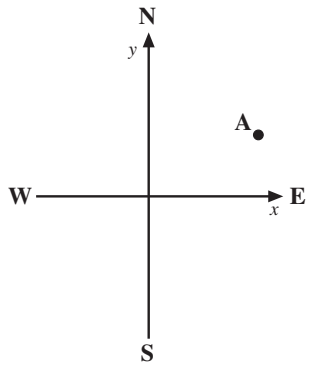
   (a)

   Plot the three points given and draw the rectangle.

   (b) Write down the coordinates of the other two corners of the rectangle.

9. A set of axes are arranged so that the x-axis runs from west to east and the y-axis from south to north.

   A ship is at the point A which has coordinates $(4, 2)$.



   (a) How far south can the ship travel before its y-coordinate becomes negative?

   (b) How far west can the ship travel before its x-coordinate becomes negative?

   (c) If the ship travels SW, how far does it travel before both coordinates become negative?

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G1.3 Plotting Straight Lines

By calculating values of coordinates you can find points and draw a graph for any relationship, such as $y = 2x - 5$.

Worked Example 1

(a) Copy and complete the following pairs of coordinates using the relationship $y = x - 2$.

$$(4,\ ?\ ),\ (1,\ ?\ ),\ (-1,\ ?\ )$$

(b) Plot the points on a set of axes.

(c) Draw a straight line through these points.

Solution

(a) For the first point $x = 4$, so

$$y = 4 - 2$$
$$= 2$$

So the first point is $(4, 2)$.

For the second point $x = 1$, so

$$y = 1 - 2$$
$$= -1$$

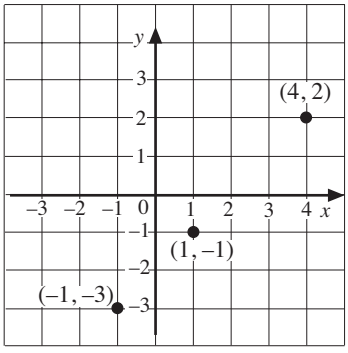
So the second point is $(1, -1)$.

For the third point $x = -1$

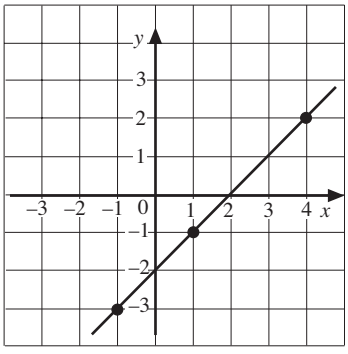
$$y = -1 - 2$$
$$= -3$$

So the third point is $(-1, -3)$.

(b) The points are plotted on these axes.



(c) The three points lie on a straight line as shown on these axes.



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Worked Example 2

Draw the graph of $y = 2x - 1$.

Solution

The first step is to find the coordinates of three points on the line. Choose any three x-values for the coordinates. Three possible values are given below.

$$(4, ?), \quad (2, ?), \quad (-2, ?)$$

Now calculate the y-values using $y = 2x - 1$.

For the first point $x = 4$, so

$$\begin{aligned} y &= 2 \times 4 - 1 \\ &= 7 \end{aligned}$$

The coordinates of this point are (4, 7).

For the second point $x = 2$, so

$$\begin{aligned} y &= 2 \times 2 - 1 \\ &= 3 \end{aligned}$$

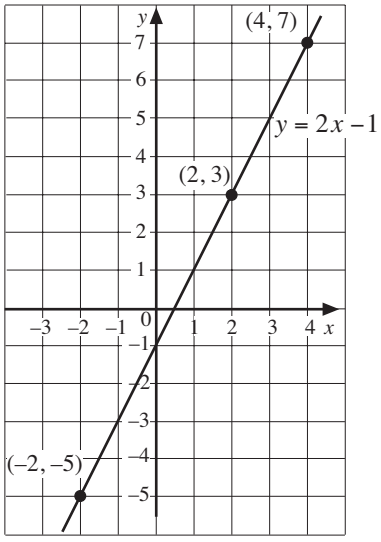
The coordinates of the second point are (2, 3).

For the third point $x = -2$, so

$$\begin{aligned} y &= 2 \times (-2) - 1 \\ &= -5 \end{aligned}$$

The third point is (–2, –5).

These points are now plotted on the axes below. Then a straight line can be drawn through them.



Note In fact, just *two* points uniquely define a straight line but it is safer to use three as a check.

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Exercises

1. (a) Copy and complete the coordinates below using the relationship $y = x + 2$.

$$(4, ?), \quad (1, ?), \quad (-3, ?)$$

(b) Draw a set of axes with x-values from –3 to 4 and y-values from –1 to 6.

(c) Plot the points with the coordinates found in (a).

(d) Check that the points lie on a straight line and draw a straight line through the points.

(e) Write down the coordinates of the point where the line crosses the y-axis.

2. (a) Use the relationship $y = 2x + 1$ to complete the coordinates below.

$$(3, ?), \quad (1, ?), \quad (-2, ?)$$

(b) Draw a set of x-axes with values from –2 to 3 and y-values from –3 to 7. Then plot the points with the coordinates obtained in (a).

(c) Draw a straight line through the points in (b).

(d) Does the line in (c) go through the point (2, 5)?

Check if these coordinates satisfy the relationship $y = 2x + 1$.

3. (a) Use the relationship $y = 3x - 2$ to complete the coordinates below.

$$(3, ?), \quad (0, ?), \quad (-2, ?)$$

(b) Plot these points on a set of axes and draw a straight line through the points.

(c) Write down the coordinates of two other points on the line. Check that they satisfy the relationship $y = 3x - 2$.

(d) Does the point (2, 5) lie on the line? Check if these coordinates satisfy the relationship $y = 3x - 2$.

4. Use a different set of axes with x-values from – 5 to 5 to draw the graph of each relationship. You will need to decide what y-values to use for each graph.

(a) $y = x + 3$

(b) $y = x - 5$

(c) $y = 3x$

(d) $y = 2x + 2$

(e) $y = 3x - 1$

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5. A teacher uses the relationship $p = 4m$ to convert the marks obtained on a test to percentages. Here m is the mark and p is the percentage.

(a) Complete a copy of the table below.

m	0	10	25
p			

(b) Complete the coordinates below using the contents of your table.

(0, ?), (10, ?), (25, ?)

(c) Using m on the horizontal axis and p on the vertical axis, plot the points with the coordinates you obtained in (b).

(d) Copy the following table and then use the graph to fill in the missing entries.

	Mark	*Percentage*
Tyson	15	
Carlyle	21	
Shanice	18	
Carl		80
Winston		60

6. The relationship $F = 32 + 1.8C$ can be used to convert temperatures in degrees Celsius, °C, to temperatures in degrees Fahrenheit, °F.

(a) Complete a copy of the table below.

Temperature in °Celsius	0	20	100
Temperature in °Fahrenheit			

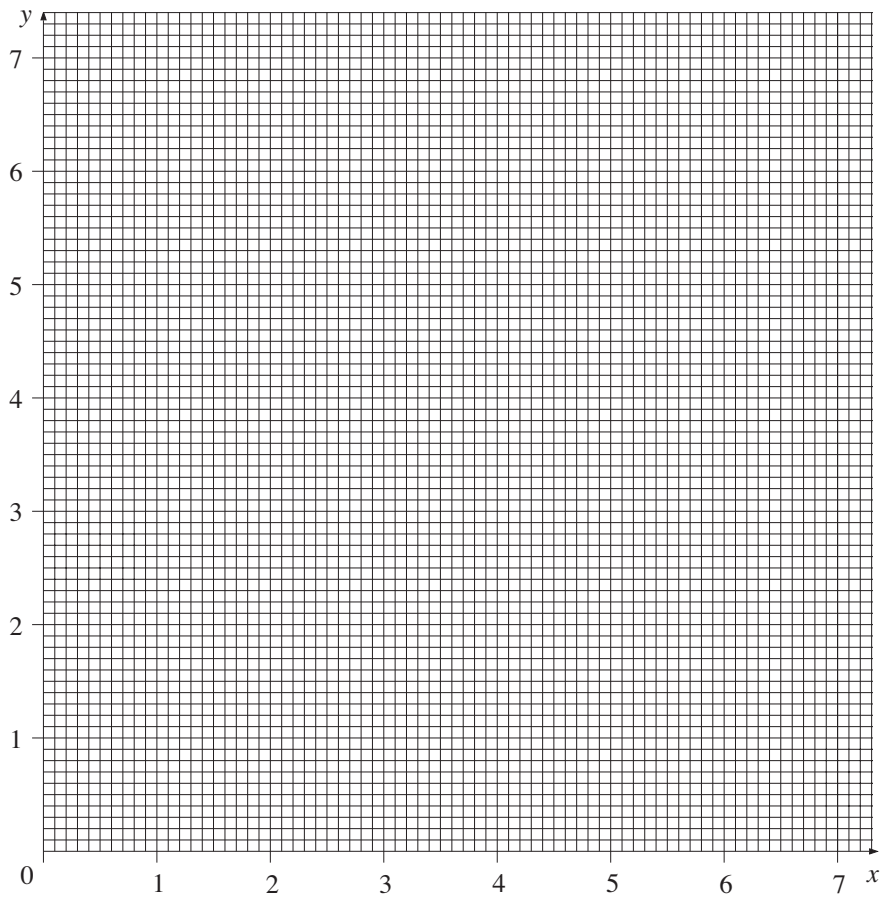
(b) Use the information in the table to draw a graph of $F = 32 + 1.8C$.

(c) The recommended temperature for a greenhouse is 80 °F. Use your graph to convert this to °Celsius.

(d) The temperature at a holiday resort is 30 °C. Use the graph to convert this to °Fahrenheit.

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7.



(a) Plot the points (0, 6), (1, 4), (2, 2) and (3, 0) on a copy of the grid above. Draw a straight line through the four points.

(b) Draw the line $y = x + 1$ on a copy of the grid above.

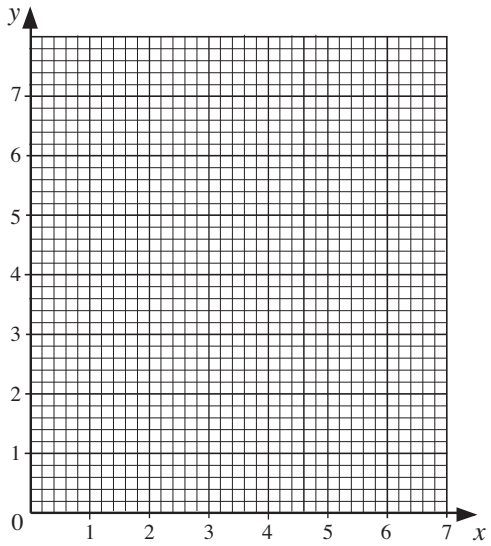
8. Copy and complete the following table for the rule.

(a) "To find *y,* double *x* and add 1."

x	*y*
0	
1	
3	

(b) Plot the values from the table onto a copy of the following coordinate grid. Join your points with a straight line.

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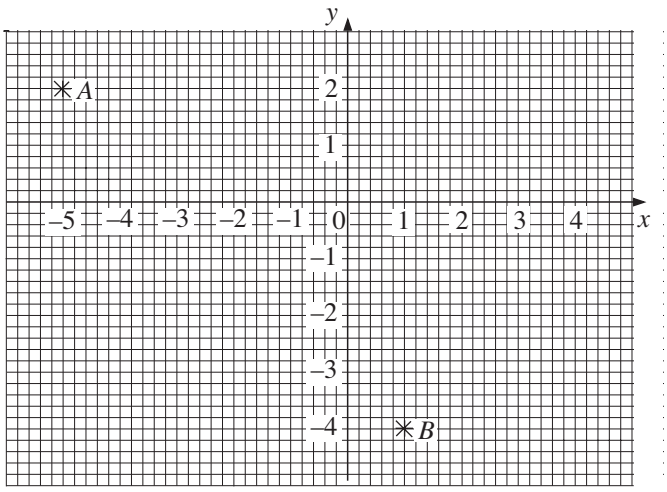
(c) Write, in symbols, the rule

"To find y, double x and add 1."

(d) Use your rule from part (c) to calculate the value of x when $y = 9$.

9. (a) Two points A and B are shown on the grid below.

Write down their coordinates.



(b) Copy and complete the table of values below for

$$y = x - 2$$

x	–2	0	2	3	4
y		–2			2

(c) Draw the graph of $y = x - 2$ on your copy of the graph above.

(d) On the grid, draw the straight line AB.

Write down the coordinates of the point where the graph of $y = x - 2$ cuts the line AB.

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G1.4 Plotting Curves

Some relationships produce *curves* rather than straight lines when plotted.



Worked Example 1

(a) Complete the table below using the relationship $y = x^2 - 2$.

x	–3	–2	–1	0	1	2	3
y							

(b) Write a list of coordinates using the data in the table.

(c) Plot the points and draw a smooth curve through them.



Solution

(a) For each value of x the y value can be calculated using $y = x^2 - 2$.

For example:

If $x = 3$ then $y = 3^2 - 2$

$= 9 - 2$

$= 7$

If $x = 1$ then $y = 1^2 - 2$

$= 1 - 2$

$= -1$

If $x = -2$ then $y = (-2)^2 - 2$

$= 4 - 2$

$= 2$

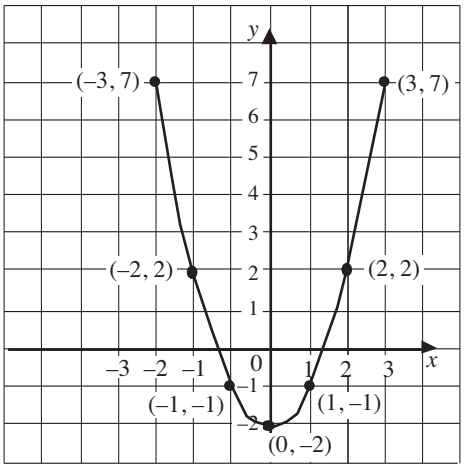
Calculating all the values gives the table.

x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7

(b) The coordinates of the points to plot are $(-3, 7), (-2, 2), (-1, -1), (0, -2), (1, -1), (2, 2)$ and $(3, 7)$.

(c) These points are plotted on the following graph and have been joined by a smooth curve.

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Worked Example 2

Draw the graph of $y = x^3 - 4x$ for values of x from -3 to 3.

Solution

The first step is to draw up and complete a table of values using the relationship

$y = x^3 - 4x$, as below.

x	–3	–2	–1	0	1	2	3
y	–15	0	3	0	–3	0	15

For example:

If $x = -3$ then $y = (-3)^3 - 4 \times (-3)$

$= -27 + 12$

$= -15$

If $x = 2$ then $y = 2^3 - 4 \times 2$

$= 8 - 8$

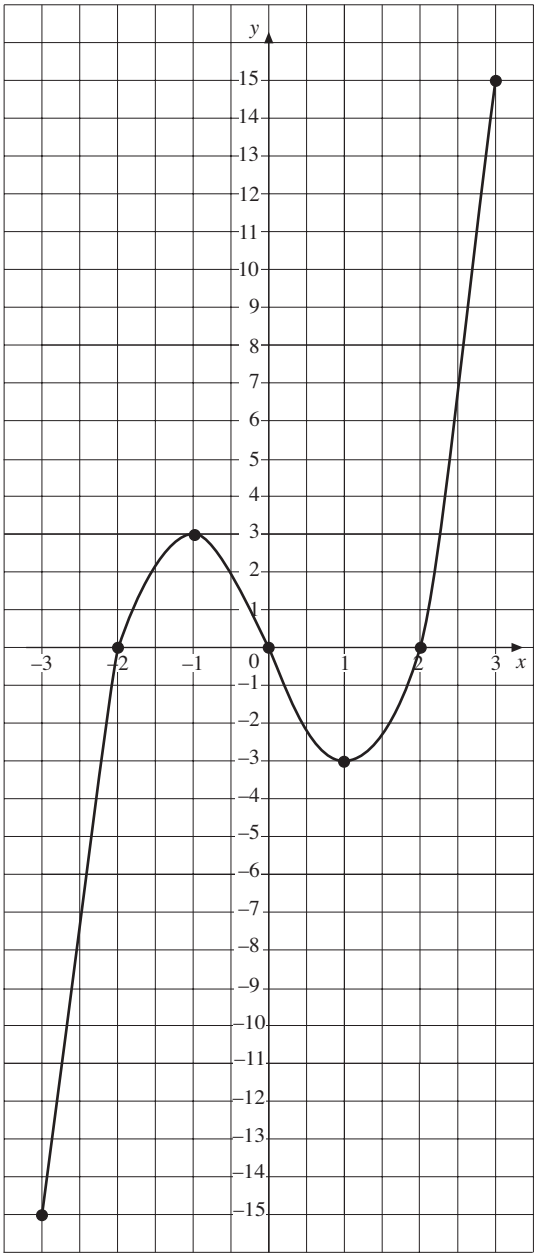
$= 0$

Each pair of values can be written as coordinates,

$$(-3, -15),\ (-2, 0),\ (-1, 3),\ (0, 0),\ (1, -3),\ (2, 0),\ (3, 15)$$

These can then be plotted and a smooth curve drawn through the points as shown in the following graph.

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Exercises

1. (a) Complete a copy of the table below for $y = x^2 - 5$.

x	–3	–2	–1	0	1	2	3
y			–4				4

(b) Write a list of coordinates. Plot these points and draw a smooth curve through the points.

2. (a) Complete a copy of the table below using the relationship $y = x^2 - 2x$.

x	–2	–1	0	1	2	3	4
y		3					8

(b) Plot the graph of $y = x^2 - 2x$ using the data in the table.

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3. (a) Complete a copy of the table and draw the graph of $y = x^2 - x - 2$.

x	–2	–1	0	1	2	3
y		0		–2		

(b) What is the value of x at the lowest point of the curve?

(c) Use your answer to (b) to calculate the corresponding value of y.

4. (a) Complete a copy of the table using the relationship $y = x^3 - x$.

x	–2	–1	0	1	2
y		0			6

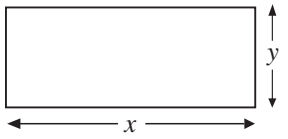
(b) Using the information in the table, sketch the graph of $y = x^3 - x$.

(c) Complete the following pairs of coordinates.

$$(-0.5,\ ?\ ),\quad (0.5,\ ?\ )$$

(d) Check that your graph passes through the points with the coordinates calculated in (c).

5. A manufacturer of postcards decides to experiment with cards of different shapes. The cards should be rectangles with an area of 120 cm^2. The height of the cards is y cm and the width is x cm.



(a) Explain why $y = \dfrac{120}{x}$.

(b) Complete a copy of the table.

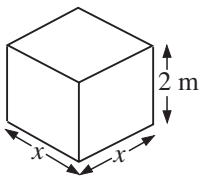
x	5	10	15	20	25	30
y		12				4

(c) Draw a graph of $y = \dfrac{120}{x}$.

(d) Use your graph to find the width of a postcard that has a height of 7 cm.

(e) If the height of a postcard must be no greater than 14 cm, what is the least width it can have?

6. Some water tanks have square bases and a height of 2 m.



(a) Explain why the volume, V, of a tank is $2x^2$.

(b) Complete a copy of the table and draw the graph of $V = 2x^2$.

x	0	0.5	1.0	1.5	2.0	2.5
V						12.5

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(c) If the base of a tank is a square sheet of metal 2.2 m by 2.2 m, find from the graph the volume of the tank.

(d) What should be the size of the base in order to give a volume of

(i) $10\ m^3$ (ii) $5\ m^3$?

7. The height, h, in metres, of the distance travelled by a ball hit straight up into the air is given by

$$h = 18t - 5t^2$$

where t is the time in seconds.

(a) Complete a copy of this table and draw a graph of h against t.

t	0	1	2	3	4
h			16		

(b) Use your graph to estimate when the ball hits the ground.

(c) What is the maximum height reached by the ball?

8. (a) Use the equation $y = x + 3$ to complete the table of values.

x	–3	–2	–1	0	1	2	3
y	0			3		5	

(b) Use the equation $y = x^2$ to complete the table of values.

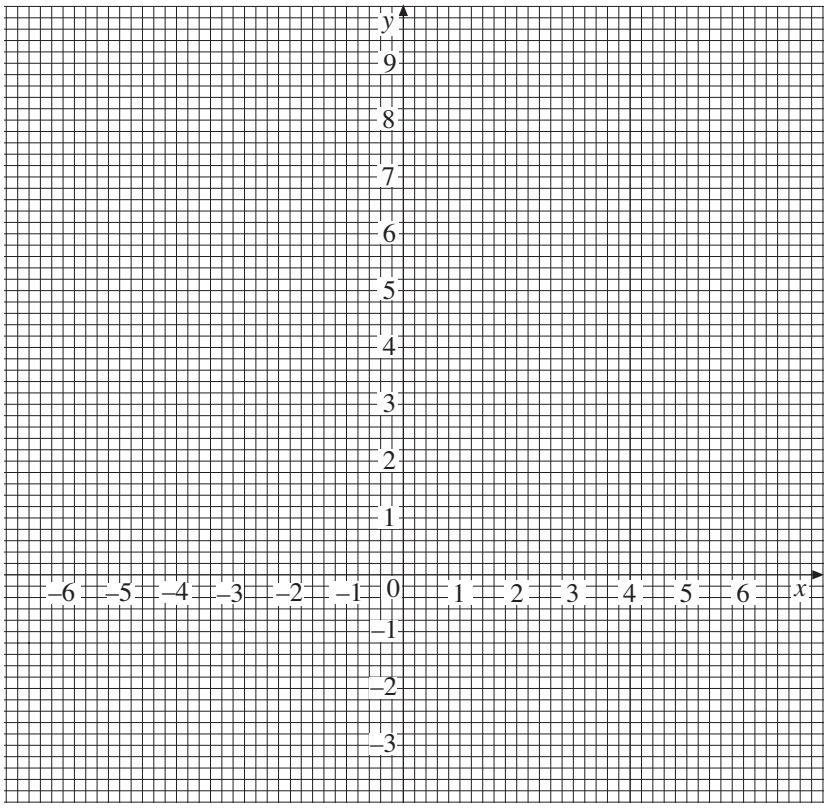
x	–3	–2	–1	0	1	2	3
y						4	

(c) Draw the graphs of

$$y = x + 3 \text{ and } y = x^2$$

on a copy of the grid below.

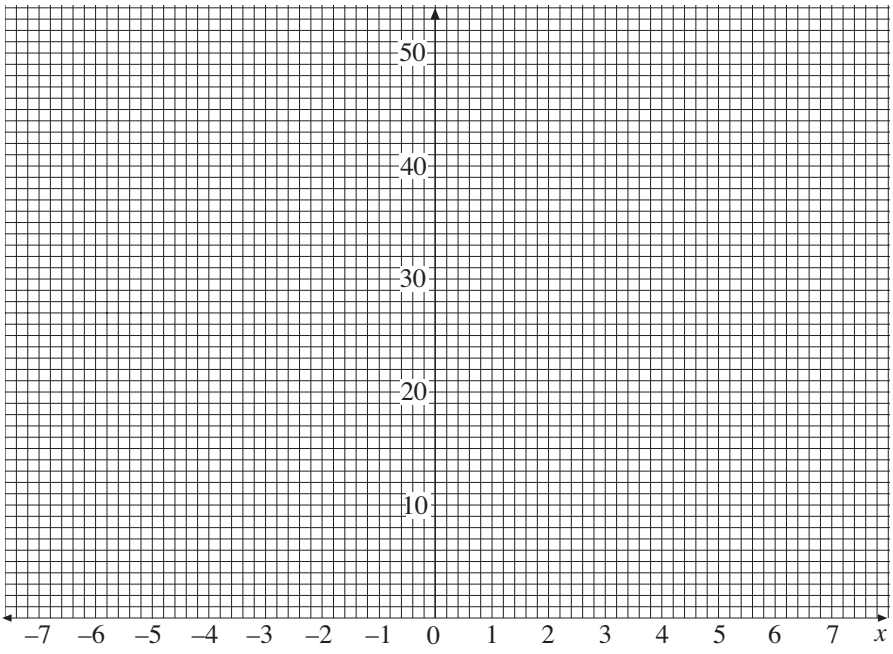
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9. (a) Use the formula $y = x^2$ to complete a copy of the table.

x	–7	–6	–5	– 4	–3	–2	–1	0	1	2	3	4	5	6	7
y	49	36		16	9	4	1	0	1	4	9	16			49

(b) Use your table of values to draw the graph of $y = x^2$.



(c) Use your graph to find the values of x when $y = 20$.

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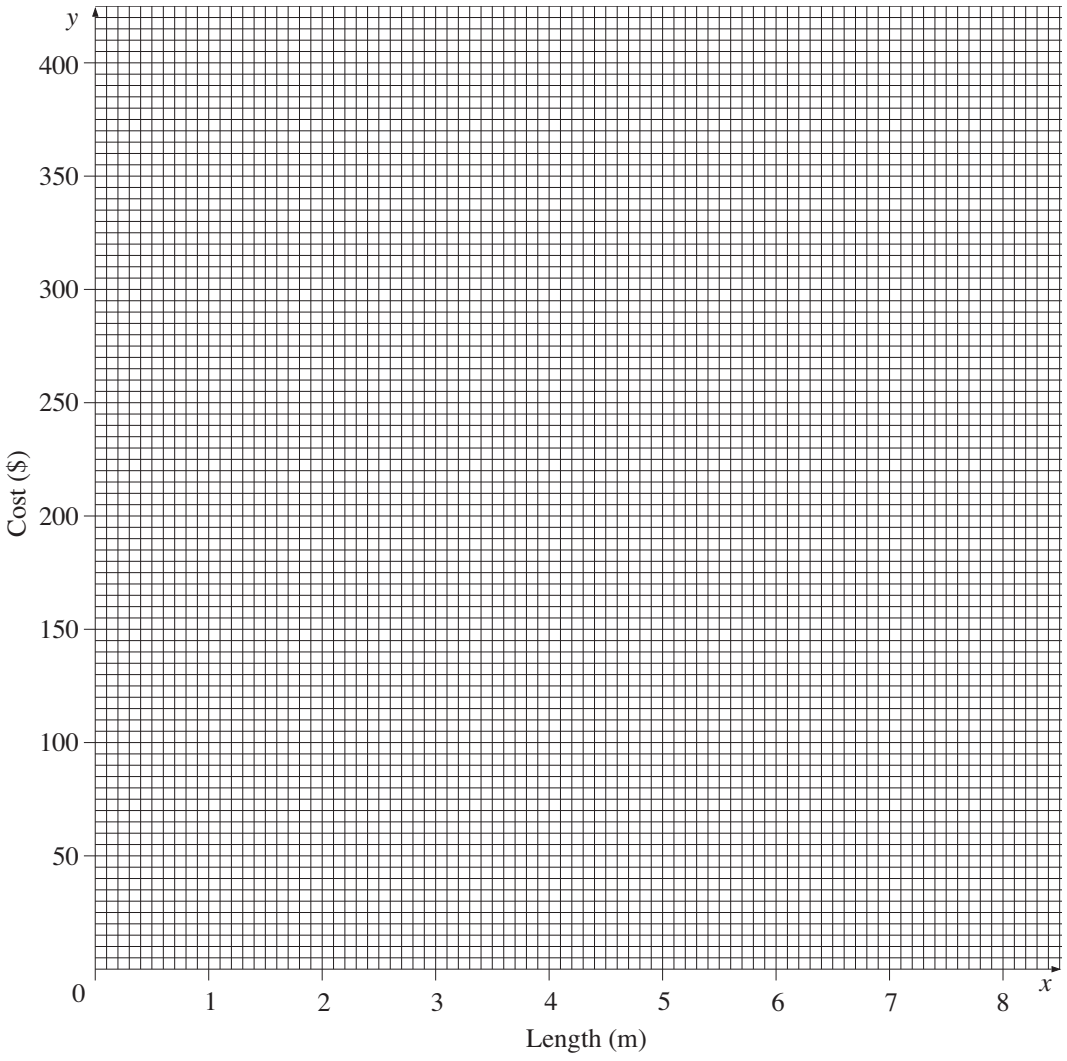
10. The annual cost of the heat lost through a wall depends on the length of the wall. When the wall is a square of length x m the annual cost, £y, is given by the equation $y = 5x^2$.

(a) Calculate the cost, £y, when x is 8 m.

(b) The table shows the cost, £y, for different values of x m.

Length, x (m)	3	4	5	6	7
Cost, y (£)	45	80	125	180	245

Use the table of values to draw the graph of $y = 5x^2$ on a copy of the grid below.

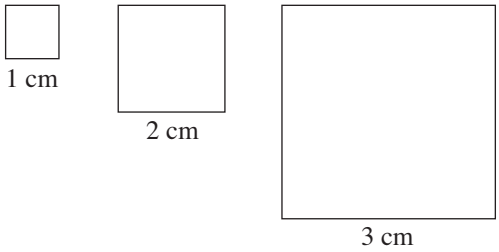


(c) The annual cost of the heat lost through a square wall is £150.

Use your graph to estimate the length of the wall

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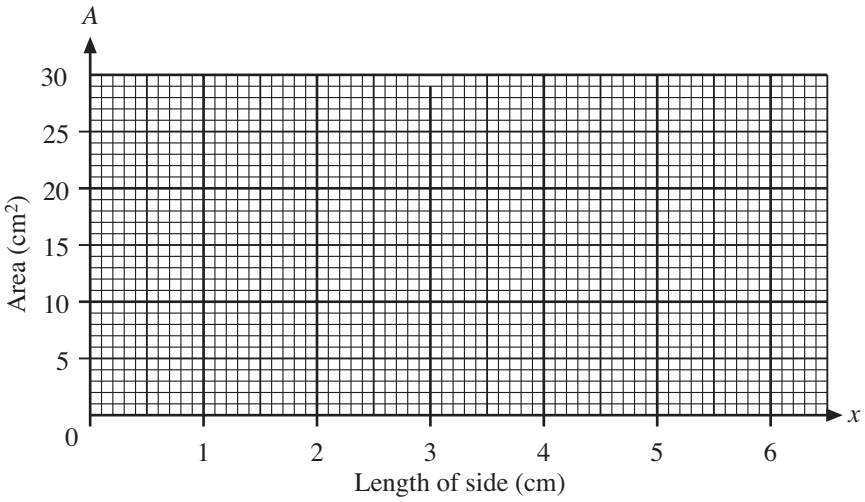
11. Angela is investigating the area of squares.



She makes a table.

Length of side (cm)	0	1	2	3	4	5
Area (cm^2)	0	1	4	9	16	25

(a) Draw a graph of area against length of side.



(b) The side of a square is x cm. The area of a square is A cm^2. Write down the formula which may be used to calculate the area from the length of the side.

(c) A square has an area of 12 cm^2. Angela wants to use the graph to find the length of the side of this square.

She draws a line on the graph to help her do this.

(i) Draw this line.

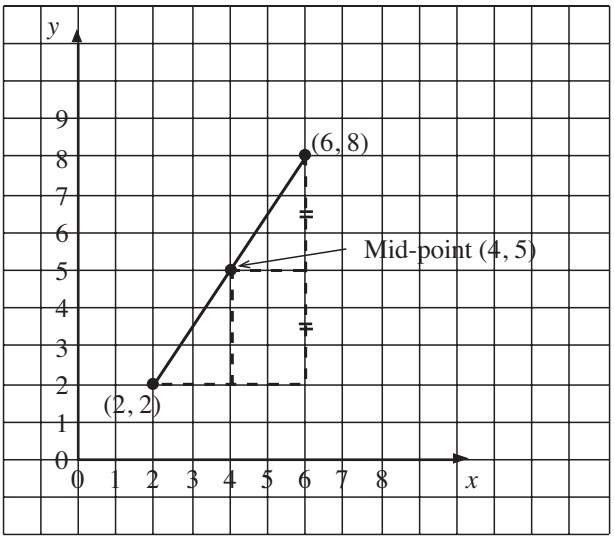
(ii) Write down the length of the side of the square whose area is 12 cm^2.

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G1.5 Mid-Points of Line Segments

The coordinates of the mid-point between two other points may be found by drawing or by calculation.

Consider the line segment that joins the point A which has coordinates (2, 2) and the point B (6, 8). The mid-point of the line segment AB is shown in the diagram below.



The value of the *x*-coordinate of the mid-point of the line segment AB is the mean value of the two *x*-coordinates of the end points A and B.

Similarly for the *y*-coordinate of the mid-point, it is the mean of the *y*-coordinates of the end points A and B.

The coordinates of the mid-point could have been calculated directly as shown below.

$$\left(\frac{2+6}{2}, \frac{2+8}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right)$$

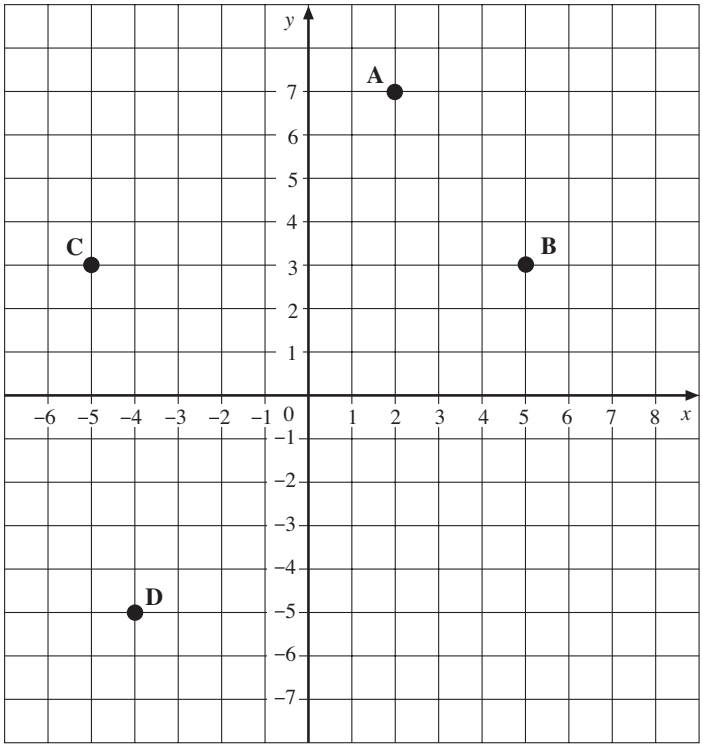
$$= (4,5)$$

Generally, for any two points, the coordinates of the mid-point of the line segment joining the points (a, b) and (c, d) is given by $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

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Worked Example 1

The diagram shows the points A, B, C and D.



Find the coordinates of the mid-point of the line segment:

(a) AB (b) AC (c) BD

Solution

(a) The coordinates of A are (2, 7).

The coordinates of B are (5, 3).

$$\text{The coordinates of the mid-point of AB} = \left(\frac{2+5}{2}, \frac{7+3}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{10}{2}\right)$$

$$= (3.5,\ 5)$$

(b) The coordinates of C are (–5, 3).

$$\text{The coordinates of the mid-point of AC} = \left(\frac{2+(-5)}{2}, \frac{7+3}{2}\right)$$

$$= \left(\frac{-3}{2}, \frac{10}{2}\right)$$

$$= (-1.5,\ 5)$$

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(c) The coordinates of D are $(-4,\ -5)$.

The coordinates of the mid-point of BD $= \left(\dfrac{5+(-4)}{2},\ \dfrac{3+(-5)}{2}\right)$

$= \left(\dfrac{1}{2},\ \dfrac{-2}{2}\right)$

$= (0.5,\ -1)$



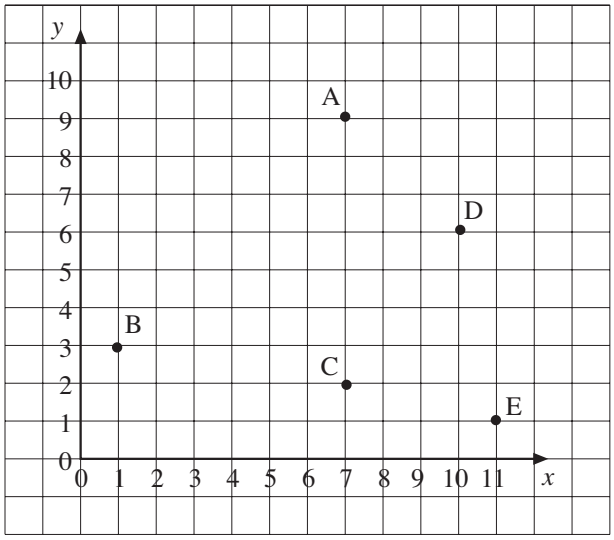
Exercises

1. (a) Draw a set of axes and mark on them the points A and B which have coordinates (1, 4) and (7, 6).

   (b) Draw the line segment AB and mark its mid-point.

   (c) Write down the coordinates of the mid-point of AB.

2. The diagram shows the points A, B, C, D and E.



   Find the coordinates of the mid-point of the line segment:

   (a) AB (b) AC (c) AD (d) AE

   (e) BE (f) CD (g) DE (h) CE

3. Determine the coordinates of the mid-point of the line segment joining the two points given in each case.

   (a) (4, 7) (8, 11) (b) (6, 2) (18, 8)

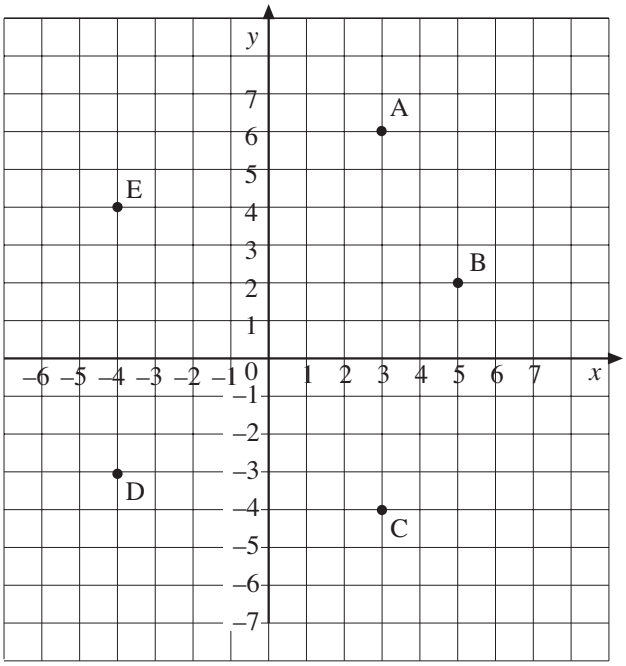
   (c) (3, 2) (9, 4) (d) (6, 3) (10, 11)

   (e) (4, 1) (3, 4) (f) (6, 6) (1, 7)

   (g) (2, 15) (13, 2) (h) (24, 2) (13, 3)

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4. The diagram shows the points A, B, C, D and E.



Determine the coordinates of the mid-point of the line segments below.

(a) AB (b) AE

(c) BD (d) BC

(e) AC (f) DC

(g) DE (h) CE

5. Determine the coordinates of the mid-point of the line segment joining each pair of points listed below.

(a) (2, 3) (4, –6) (b) (–2, 1) (3, –6)

(c) (–2, –3) (–8, –10) (d) (–2, 4) (5, –6)

(e) (–3, –2) (4, 7) (f) (6, –2) (8, –10)

(g) (2, –6) (7, –5) (h) (8, –3) (–10, –11)

6. The diagram shows a cuboid.

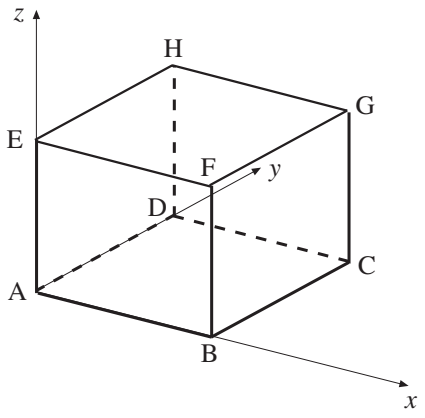
The coordinates of the vertices are listed below.

A (0, 0, 0) B (7, 0, 0)

C (7, 6, 0) D (0, 6, 0)

E (0, 0, 5) F (7, 0, 5)

G (7, 6, 5) H (0, 6, 5)



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(a) Explain why the coordinates of the mid-point of DG are (3.5, 6, 2.5).

(b) Determine the coordinates of the mid-point of each of the following line segments.

(i) AB

(ii) CD

(iii) DH

(iv) FG

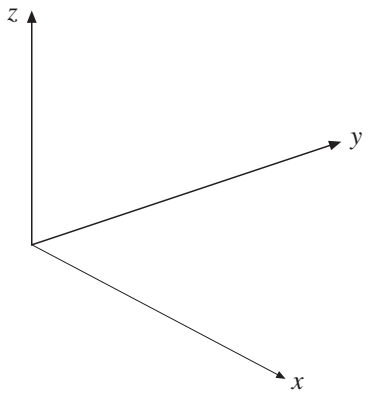
7. The points A, B and C have coordinates (4, 2, 1), (6, 10, 7) and (4, 8, 11).

(a) Determine the coordinates of the mid-points of AB and AC.

(b) The mid-point of AB is joined to the mid-point of AC by a line segment. Determine the coordinates of the mid-point of this line segment.

G1.6 Three-Dimensional Coordinates

Coordinates can also be applied to three dimensions. Three numbers are required to identify the position of a point in space.



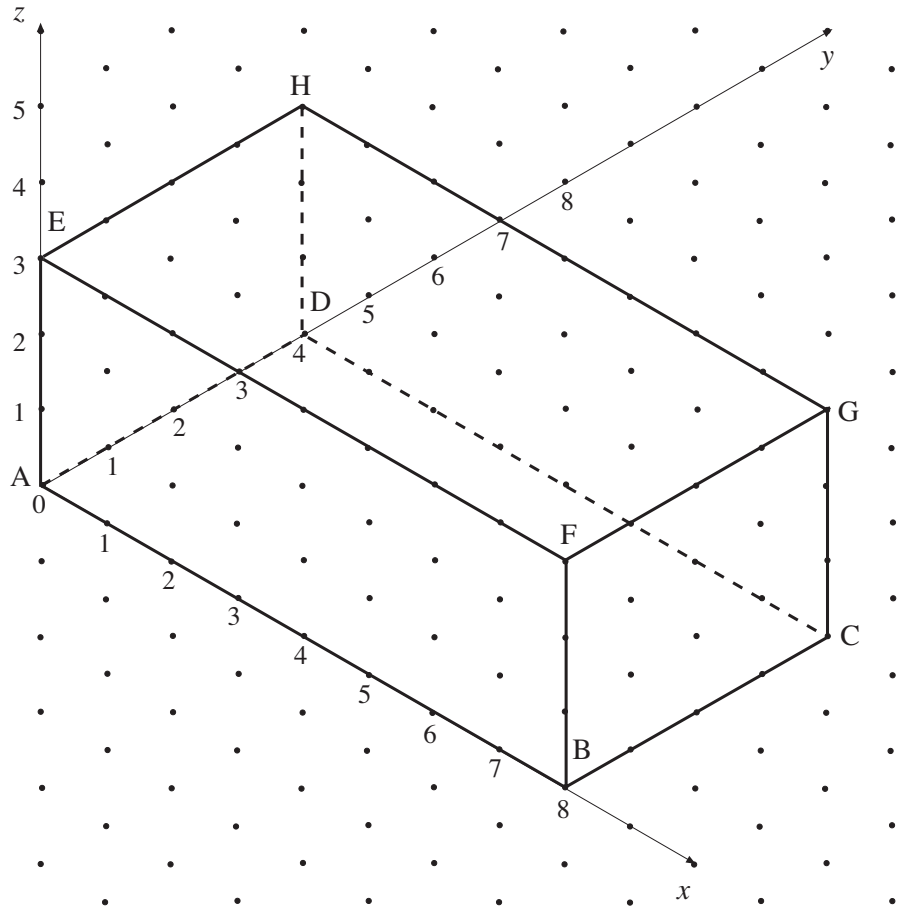
The coordinates are then written as ordered triples in the form of (x, y, z).

Worked Example 1

The following diagram shows a cuboid.

Write down the coordinates of each vertex of the cuboid.

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Solution

A (0, 0, 0)

B (8, 0, 0)

C (8, 4, 0)

D (0, 4, 0)

E (0, 0, 3)

F (8, 0, 3)

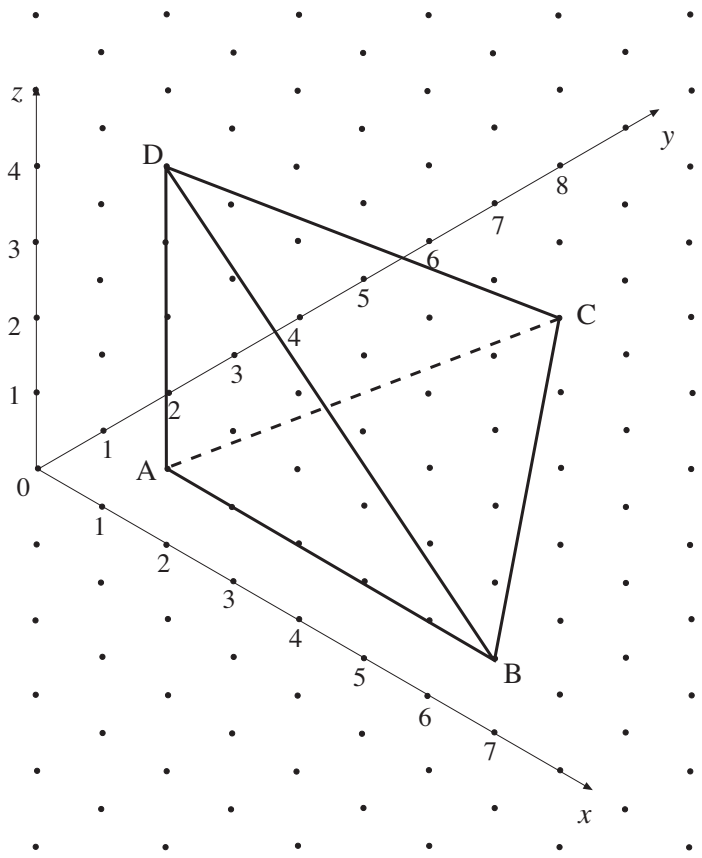
G (8, 4, 3)

H (0, 4, 3)

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Worked Example 2

The diagram shows a triangular-based pyramid, ABCD, in which D is vertically above A. The base ABC is horizontal. Write down the coordinates of each of the vertices.



Solution

A (1, 1, 0)

B (6, 1, 0)

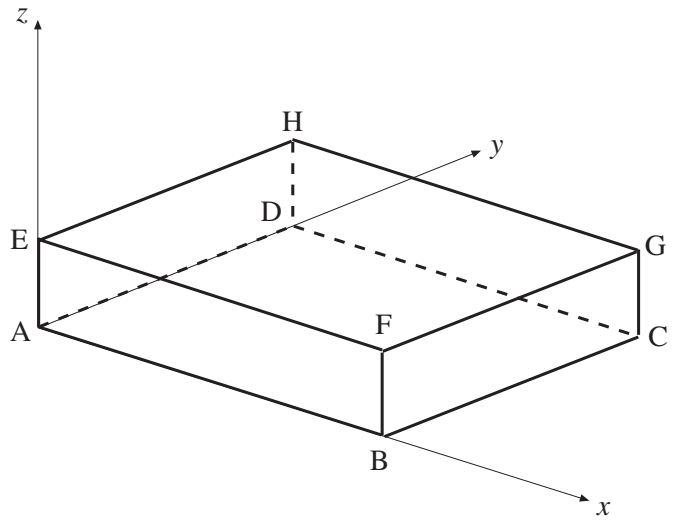
C (2, 6, 0)

D (1, 1, 4)

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Worked Example 3

The diagram shows a cuboid and the x, y and z axes.



If $AB = 6$, $AD = 5$ and $AE = 2$, write down the coordinates of each vertex of the cuboid.

Solution

A $(0, 0, 0)$ B $(6, 0, 0)$ C $(6, 5, 0)$

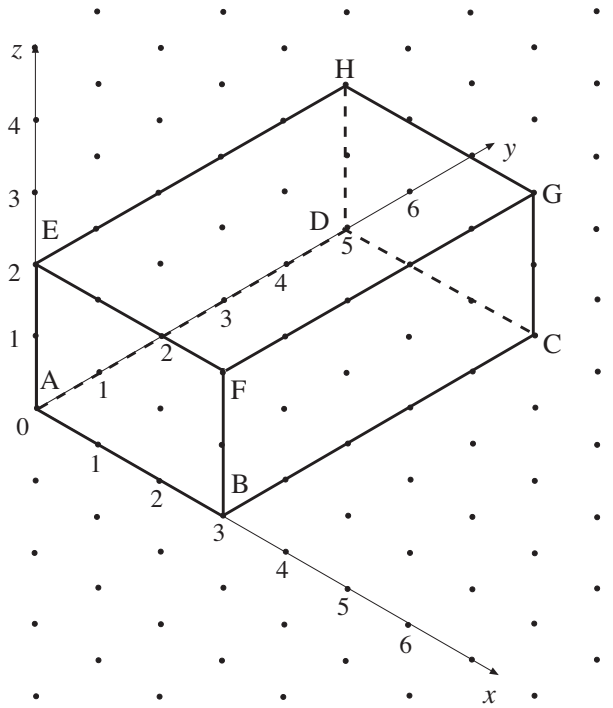
D $(0, 5, 0)$ E $(0, 0, 2)$ F $(6, 0, 2)$

G $(6, 5, 2)$ H $(0, 5, 2)$

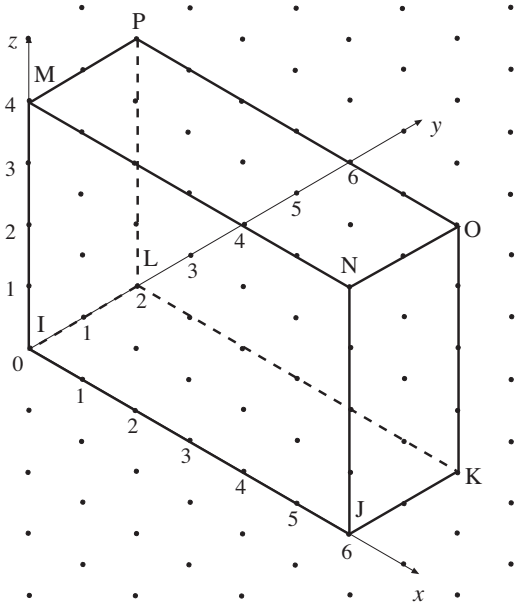
Exercises

1. Write down the coordinates of each of the vertices of the following cuboids.

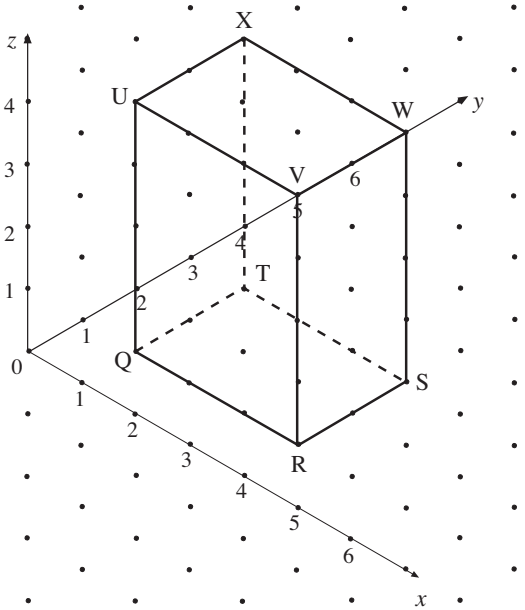
(a)



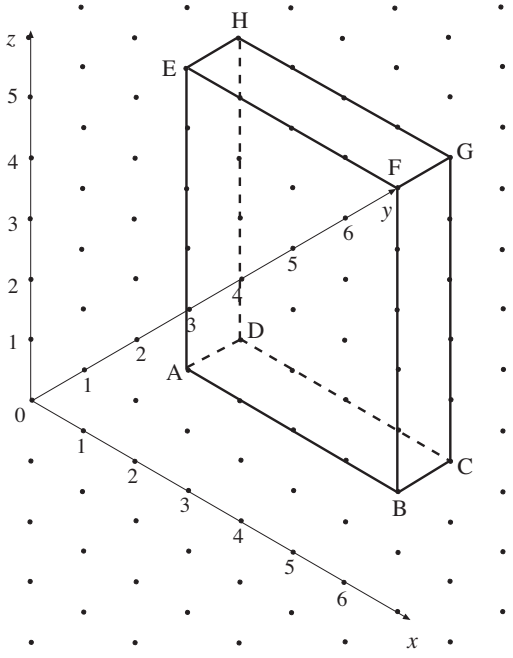
(b)



(c)



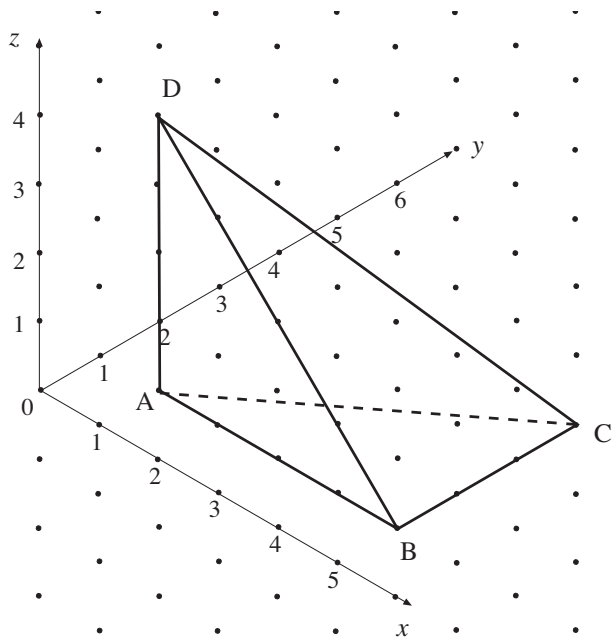
(d)



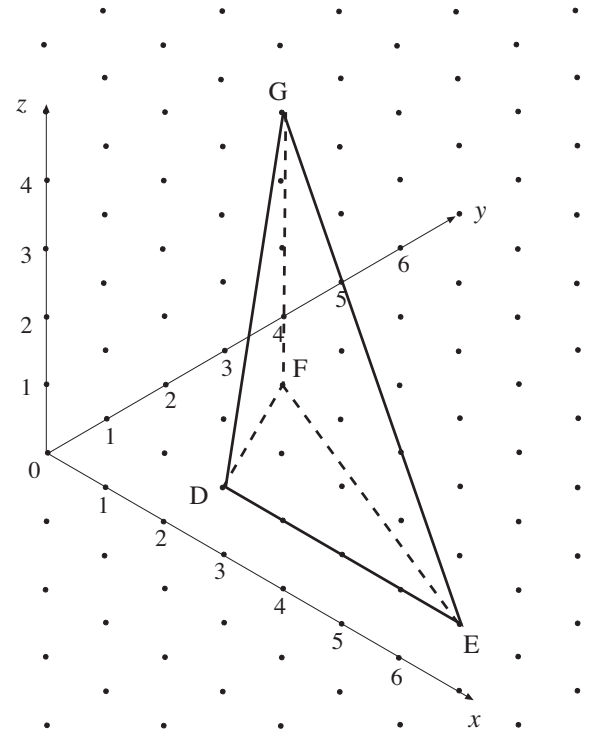
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2. Write down the coordinates of each of the vertices of the following solids.

(a)

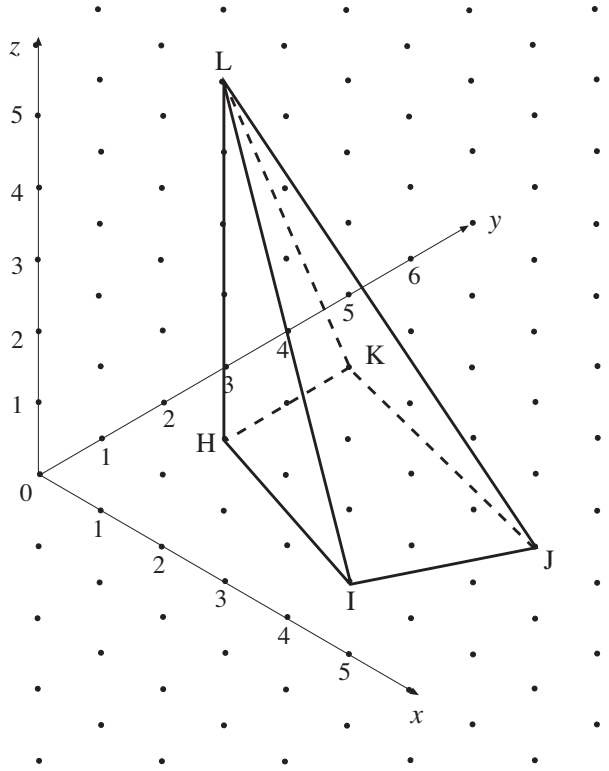


(b)

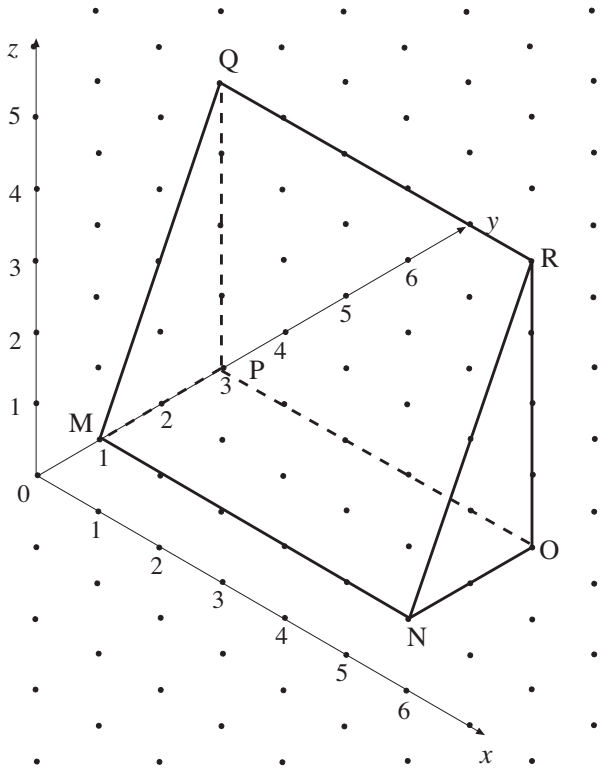


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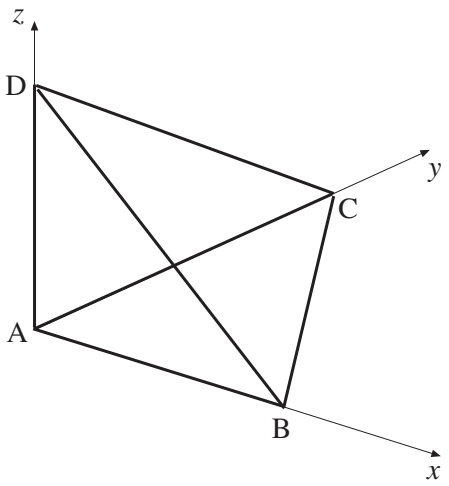
(c)



(d)

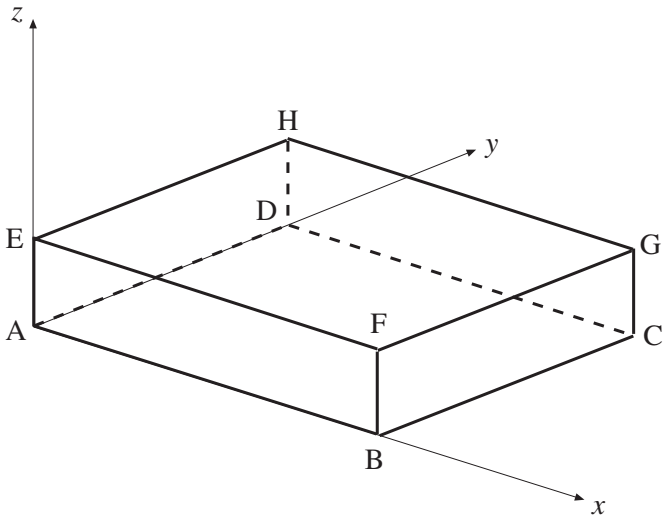


3. The diagram shows a tetrahedron.



If $AB = 3$, $AC = 4$ and $AD = 5$, write down the coordinates of each vertex of the tetrahedron.

4. The diagram shows a cuboid.

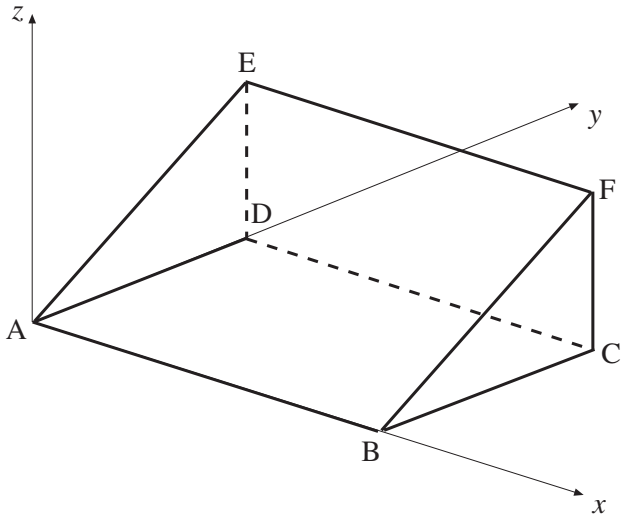


The coordinates of G are $(7, 6, 3)$.

Write down the coordinates of the other vertices.

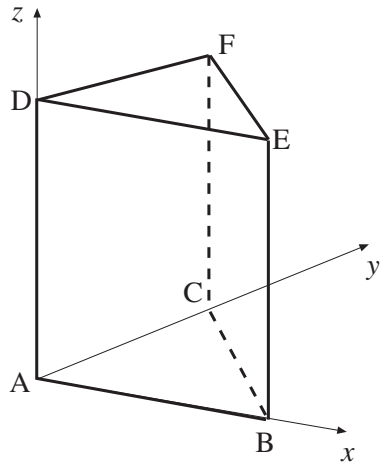
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5. In the diagram below, the x- and y-axes are horizontal and the z-axis is vertical. The diagram shows a triangular prism with a horizontal base, ABCD. The faces of the prism, AED and BFC, are vertical.



If $AB = 7$, $AD = 4$ and $DE = 5$, write down the coordinates of each vertex of the prism.

6. In the diagram below, the x- and y-axes are horizontal and the z-axis is vertical. The diagram shows a triangular prism with horizontal faces, ABC and DEF. The edges BE and CF are vertical.



If $AB = 4$, $AC = 5$ and $AD = 8$, write down the coordinates of each vertex of the prism.

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