G3 *Using Graphs to Solve Equations* — Introduction

Learning objectives

This unit uses the concept of graphical functions in order to solve equations. After completing Unit G3 you should

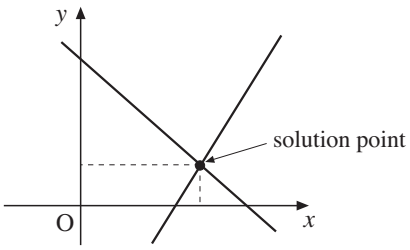
- be able to graph linear simultaneous equations in order to find the solution
- understand the graphical form of well-known functions; these include linear, quadratic, cubic and reciprocal functions
- ✻ be able to graph functions in order to solve equations, including the solution of quadratic and linear equations
- ✻ be able to construct and use curves and tangents in contexts, including the use of velocity (speed)-time and distance-time graphs.

Introduction

This is covered in Unit G1.

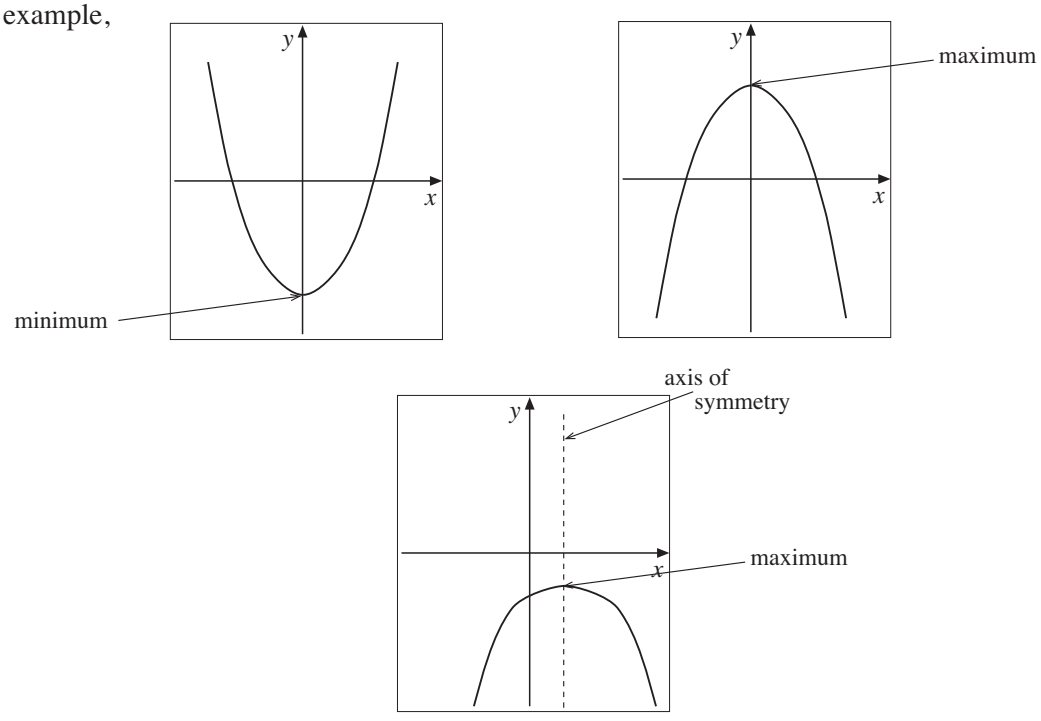
Key points and principles

- Simultaneous (linear) equations can be solved by finding the point of intersection of the two straight lines.



- Quadratic functions are parabolic in shape.

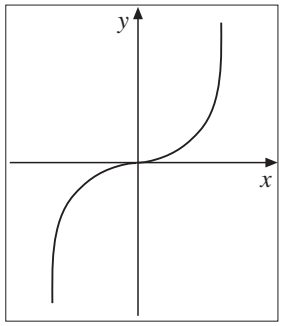
  For example,

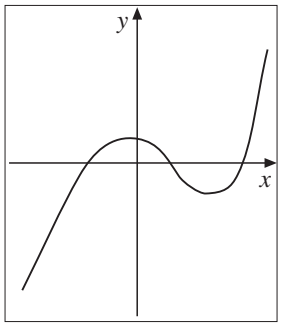


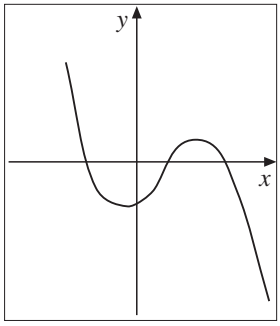
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- Cubic functions are of the form

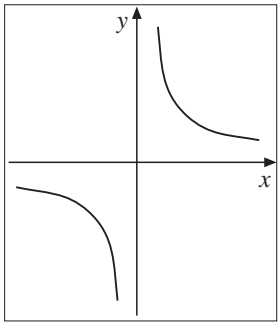


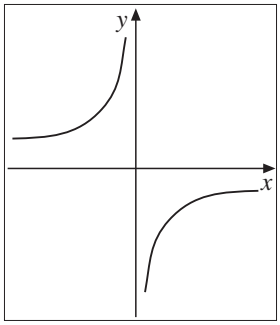
or


or


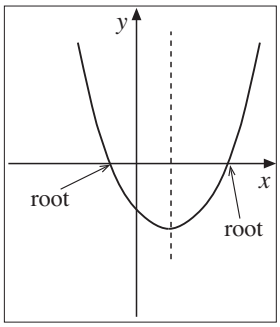
(turning point at origin)

- Reciprocal functions are of the form

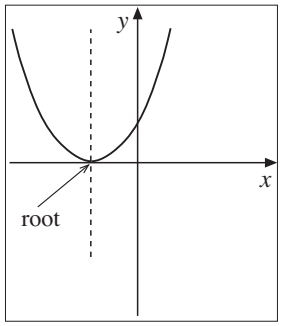


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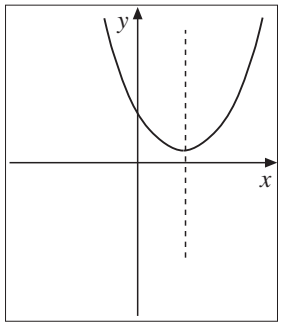
- *Roots of a quadratic equations*: there can be 2 or 1 or 0 roots for a quadratic equation.
  For example,



2 roots



1 root



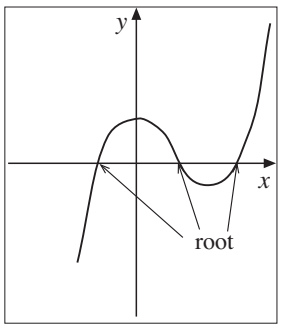
no root

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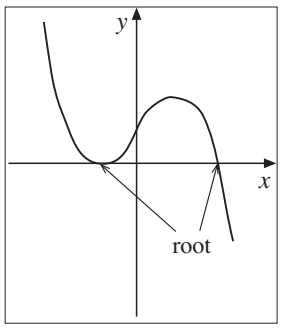
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- *Roots of a cubic equations*: there can be 3 or 2 or 1 root of a quadratic equation.

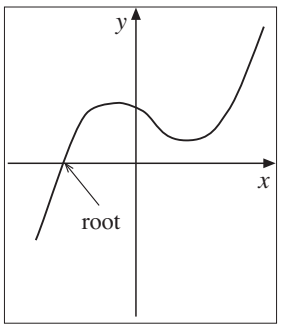
  For example,



3 roots



2 roots



1 root

- You can solve equations of the form $ax^2 + bx + c = g(x)$ when $g(x) = dx + e$ by graphing the quadratic

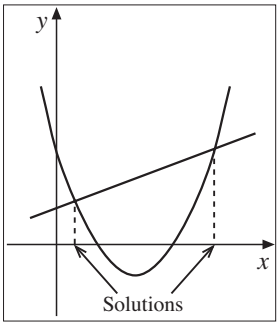
  $$f(x) = ax^2 + bx + c \quad \text{and} \quad g(x)$$

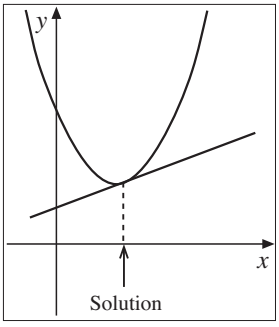
  The point(s) of intersection are the solutions.

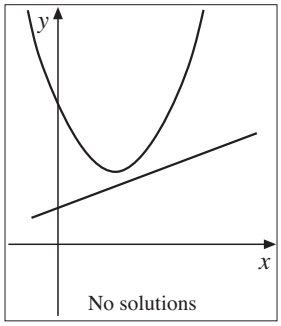
Facts to remember

- Quadratic functions are parabolic in shape (see above)
- Quadratic equations can have 2 or 1 or 0 roots.
- Cubic equations can have 3 or 2 or 1 root.
- Solving a quadratic equation and linear equation, for example $y = ax^2 + bx + c$, $y = dx + e$, can result in 2 solutions, 1 solution or no solution.

  For example,







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Glossary of terms

Simultaneous (linear) equations take the form $ax + by = c$
$dx + ey = f$
(a, b, c, d, e and f constants)

Quadratic functions are of the form $f(x) = ax^2 + bx + c \quad (a \neq 0)$

For example, $f(x) = 2x^2 - x + 1$

Quadratic equations are of the form $ax^2 + bx + c = 0 \quad (a \neq 0)$

For example, $2x^2 - x + 1 = 0$

Cubic functions are of the form $f(x) = ax^3 + bx^2 + cx + d \quad (a \neq 0)$

For example, $f(x) = x^3$, $f(x) = 2x^3 - x + 1$

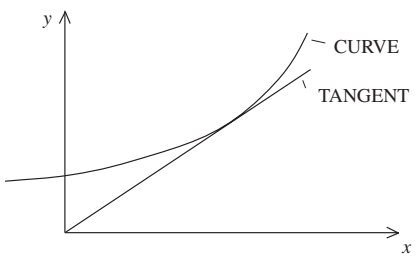
Cubic equations are of the form $ax^3 + bx^2 + cx + d = 0 \quad (a \neq 0)$

For example, $2x^3 - x + 1 = 0$

Reciprocal functions are of the form $f(x) = \frac{k}{x}$

For example, $f(x) = \frac{1}{x}$, $f(x) = -\frac{2}{x}$

Tangent a line that touches a curve at one point only, as shown opposite.



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