H1 *Angles and Symmetry* Data Sheets

Data Sheets

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H1.3 Angles in Triangles
H1.4 Angles and Parallel Lines: Results
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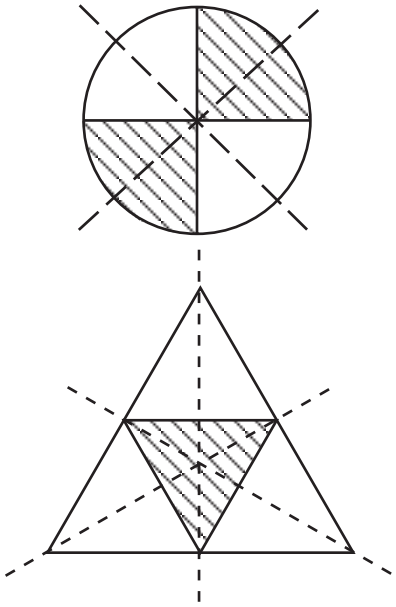
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Data Sheet H1.1 *Line and Rotational Symmetry*

An object has *rotational symmetry* if it can be rotated about a point so that it fits on top of itself without completing a full turn. The number of times this can be done is the *order* of rotational symmetry.

Shapes have *line symmetry* if a mirror could be placed so that one side of the shape is an exact reflection of the other.

Example



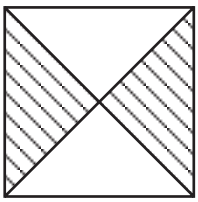
Rotational symmetry of order 2

2 lines of symmetry (shown with dotted lines)

Rotational symmetry of order 3

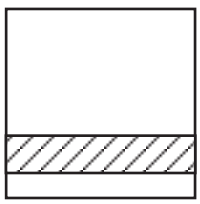
3 lines of symmetry (shown with dotted lines)

Exercises What is (a) the order of rotational symmetry,
(b) the number of lines of symmetry
of each of these shapes? Show the lines of symmetry.



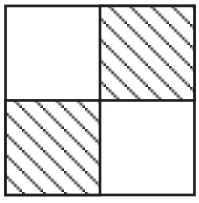
(a)

(b)



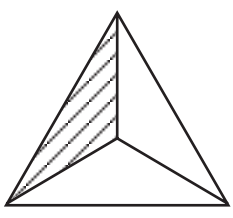
(a)

(b)



(a)

(b)



(a)

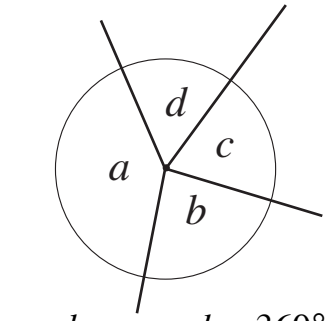
(b)

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Data Sheet H1.2

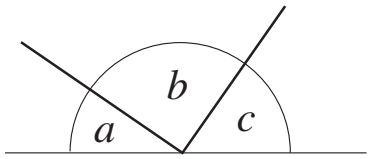
Angle Properties

1. *Angles at a Point*
   The angles at a point will always add up to 360°.
   It does not matter how many angles are formed at the point – their total will always be 360°.



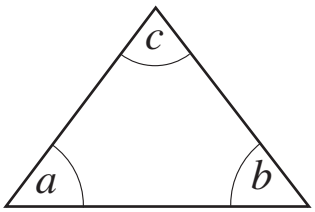
$a + b + c + d = 360°$

2. *Angles on a Line*
   Any angles that form a straight line add up to 180°.



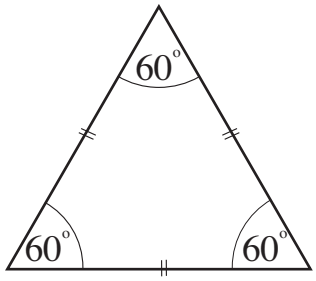
$a + b + c = 180°$

3. *Angles in a Triangle*
   The angles in any triangle add up to 180°.

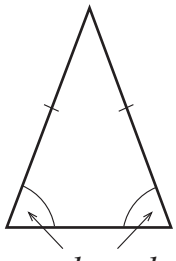


$a + b + c = 180°$

4. *Angles in an Equilateral Triangle*
   In an equilateral triangle all the angles are 60° and all the sides are the same length.

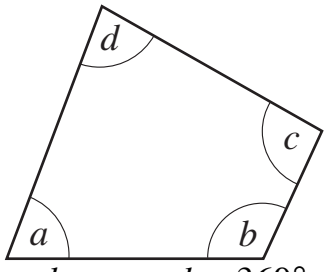


5. *Angles in an Isosceles Triangle*
   In an isosceles triangle two sides are the same length and the two angles are the same size.



equal angles

6. *Angles in a Quadrilateral*
   The angles in any quadrilateral add up to 360°.



$a + b + c + d = 360°$

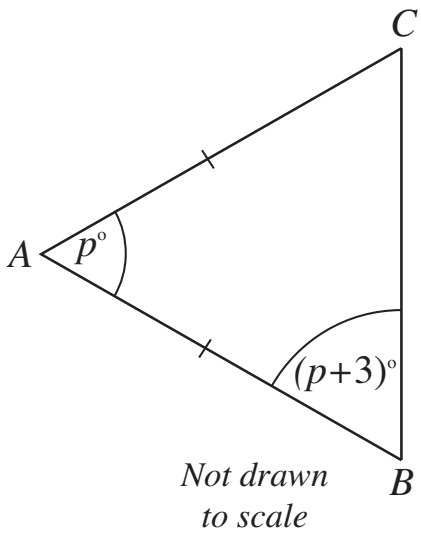
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Data Sheet H1.3 *Angles in Triangles*

Note that the angles in any triangle sum to 180°.

Example

In this figure, ABC is an isosceles triangle with $\angle CAB = p°$ and $\angle ABC = (p + 3)°$.



Not drawn to scale

(a) Write an expression in terms of p for the value of the angle at C.

(b) Determine the size of EACH angle in the triangle.

Solution

(a) As ABC is an isosceles triangles,

$A\hat{C}B =$ ☐

(b) For triangle ABC,

☐ + ☐ + ☐ = 180°

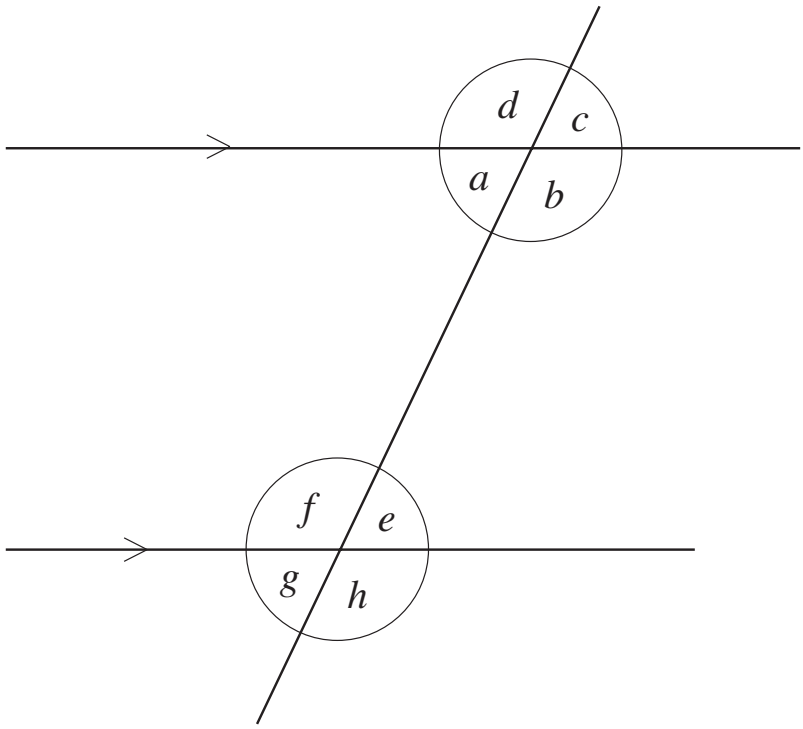
☐ p + ☐ = 180°

☐ p = ☐

p = ☐ °

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Data Sheet H1.4 *Angles and Parallel Lines: Results*



Results

- *Corresponding* angles are equal. e.g. $d = f$, $c = e$
- *Alternate* angles are equal. e.g. $b = f$, $a = e$
- *Supplementary* angles sum to 180°. e.g. $a + f = 180°$

Thus

- If *corresponding* angles are equal, then the two lines are **parallel**.
- If *alternate* angles are equal, then the two lines are **parallel**.
- If *supplementary* angles sum to 180°, then the two lines are **parallel**.

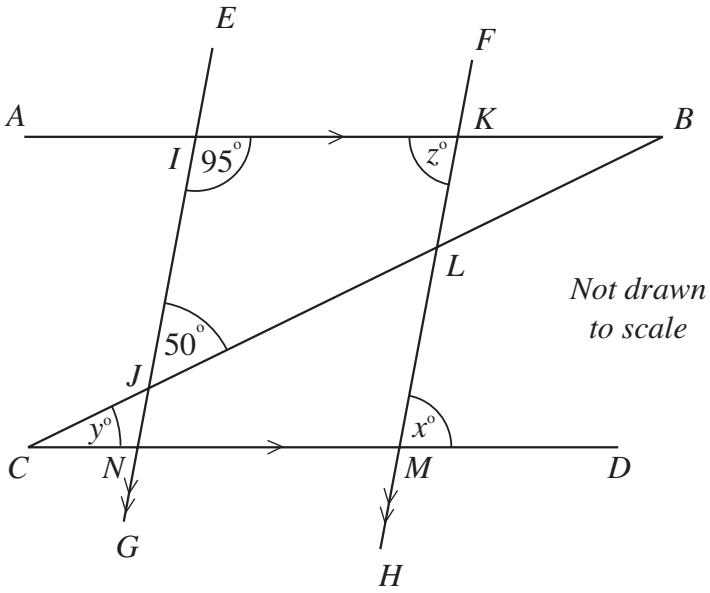
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Data Sheet H1.5 *Angles and Parallel Lines: Example*

Example

In this diagram,
AB is parallel to *CD*,
EG is parallel to *FH*,
angle *IJL* = 50° and
angle *KIJ* = 95°.

Calculate the values of *x*, *y* and *z*, showing clearly the steps in your calculations.



Solution

x Angles *BIG* and *END* are [ ] angles, so

$95° + E\hat{N}D =$ [ ]°

$E\hat{N}D =$ [ ]°

But angles *END* and *FMD* are [ ] angles, so

$x =$ [ ]°

y Angles *BCD* and *ABC* are [ ] angles, so

$y =$ [ ]

In triangle *BIJ*,

$y +$ [ ]° + [ ]° $= 180°$

So $y = 180° -$ [ ]°

$y =$ [ ]°

z Angles *AKH* and *FMD* are [ ] angles, so

$z =$ [ ]°

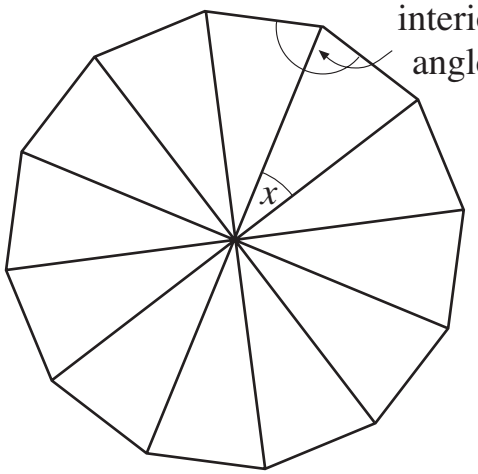
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Data Sheet H1.6 *Angle Symmetry in Regular Polygons*

Example 1 Find the interior angle of a regular dodecagon.

Solution

A regular dodecagon has ☐ sides.



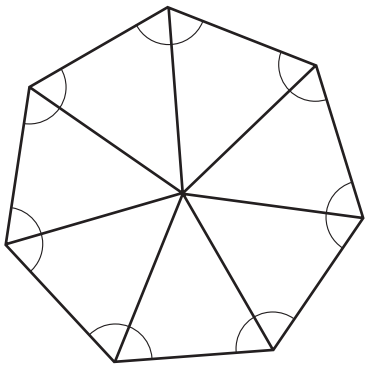
The angle, marked x, is given by

$$x = \frac{360^\circ}{\square} = \square^\circ$$

The other angle in each of the isosceles triangles is $\dfrac{(180^\circ - \square)}{2} = \square^\circ$

The interior angle is $2 \times \square^\circ = \square^\circ$

Example 2 Find the sum of the interior angles of a regular heptagon.



Solution

You can split a regular heptagon into ☐ isosceles triangles.

Each triangle contains three angles that sum to ☐°.

Total of all angles = $7 \times \square^\circ = \square^\circ$.

We need to exclude the angles round the centre that sum to ☐°.

Hence sum of interior angles = ☐° − ☐°

= ☐°

Note: Is the result the same for an *irregular* heptagon?

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