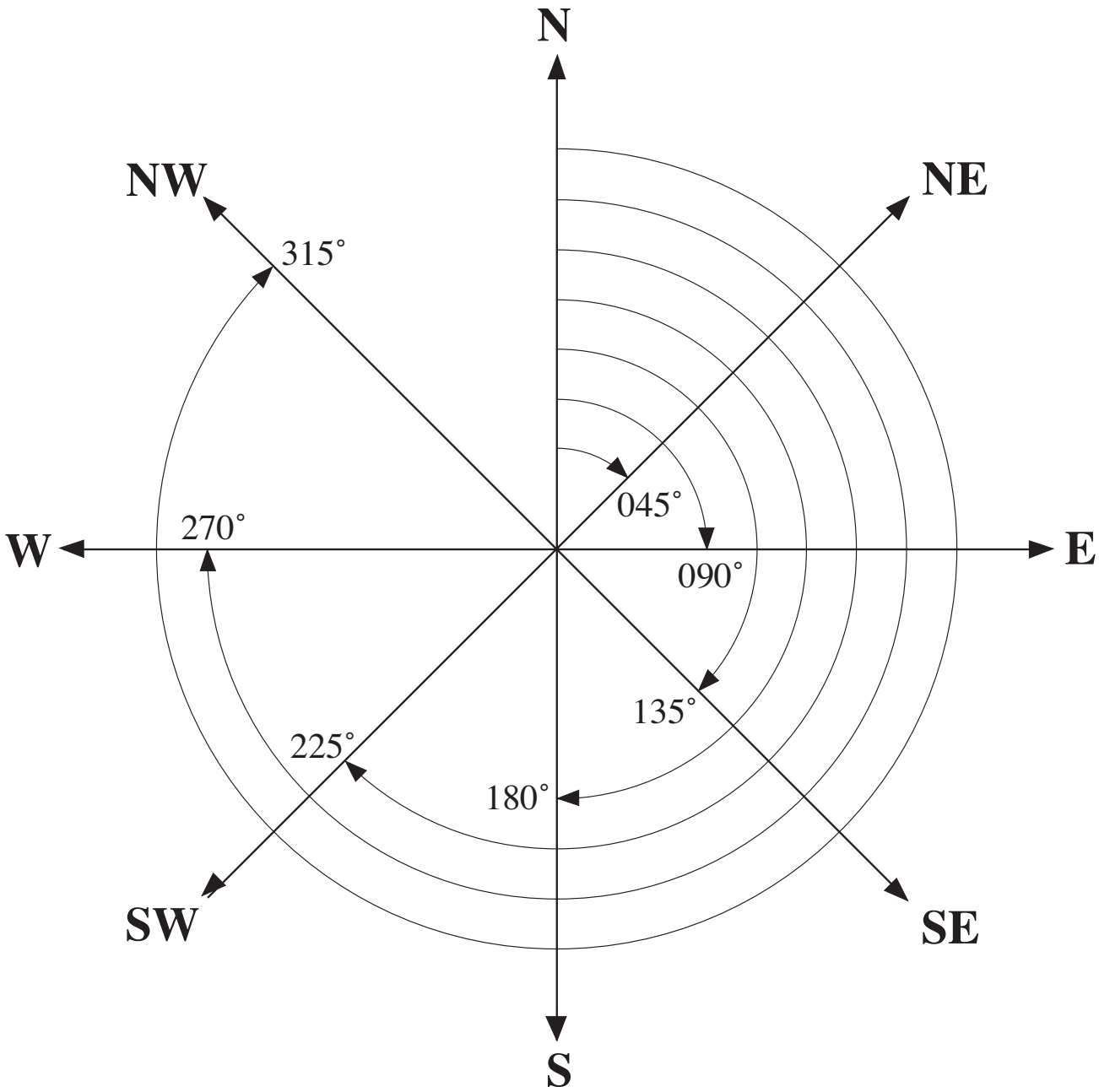
H2 *Angles, Circles and Tangents* Data Sheets

Data Sheets

H2.1 Compass Direction and Bearings
H2.2 Compass Bearings
H2.3 Angles and Circles: Results
H2.4 Angles and Circles: Example 1
H2.5 Angles and Circles: Example 2
H2.6 Angles and Circles: More Results
H2.7 Angles and Circles: More Examples
H2.8 Circles and Tangents: Results A
H2.9 Circles and Tangents: Results B
H2.10 Circles and Tangents: Example 1
H2.11 Circles and Tangents: Example 2

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Data Sheet H2.1 *Compass Direction and Bearings*



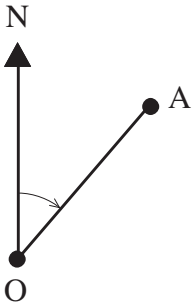
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Data Sheet H2.2 *Compass Bearings*

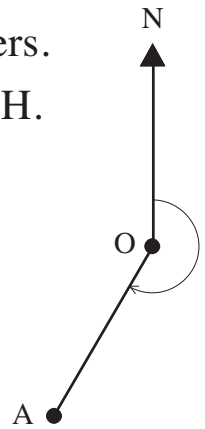
NOTES

1. Bearings are written as three-figure numbers.

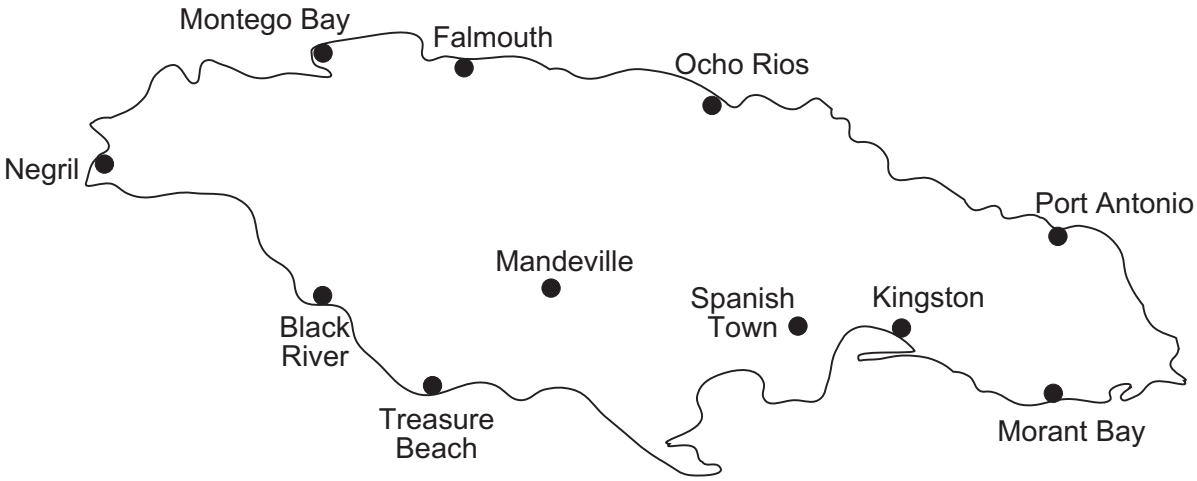
2. They are measured clockwise from NORTH.



The bearing of
A from O is 050°.



The bearing of
A from O is 210°.



Use a protractor to determine the bearing of

(a) Kingston from Montego Bay ____ °

(b) Montego Bay from Kingston ____ °

(c) Port Antonio from Kingston ____ °

(d) Spanish Town from Kingston ____ °

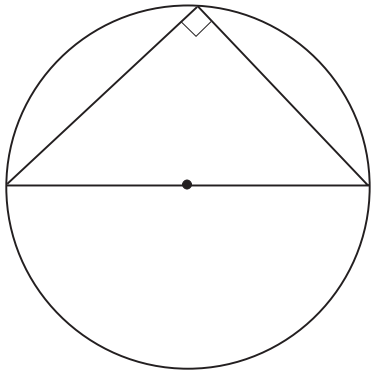
(e) Kingston from Negril ____ °

(f) Ocho Rios from Treasure Beach. ____ °

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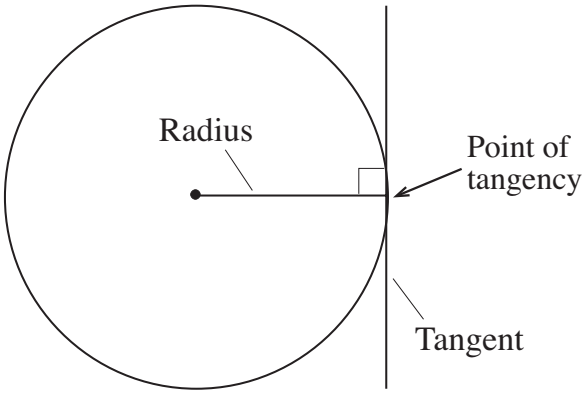
Data Sheet H2.3 *Angles and Circles: Results*

When a triangle is drawn in a semi-circle as shown, the angle on the perimeter is always a right angle.



A *tangent* is a line that touches only one point on the circumference of a circle. This point is known as the *point of tangency*.

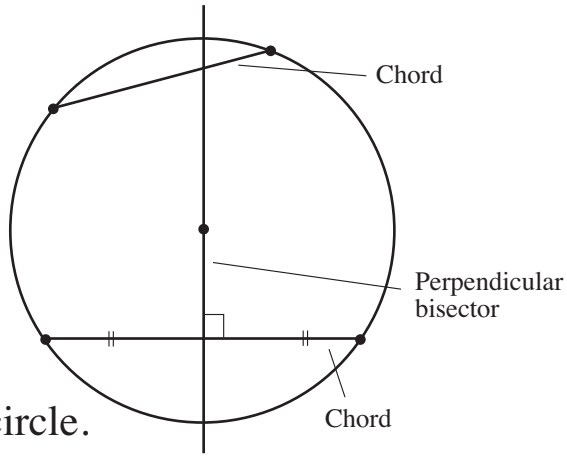
A tangent is always perpendicular to the radius.



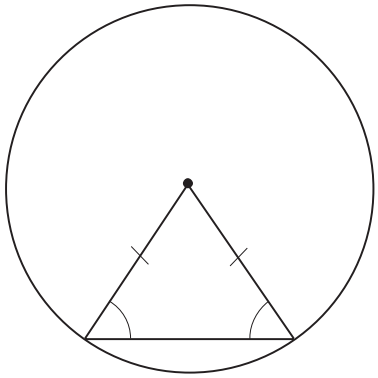
A *chord* is a line joining any two points on the circle.

The *perpendicular bisector* is a second line that divides the first line in half and is at right angles to it.

The perpendicular bisector of a chord always passes through the centre of the circle.



When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed, so the two angles marked are equal.

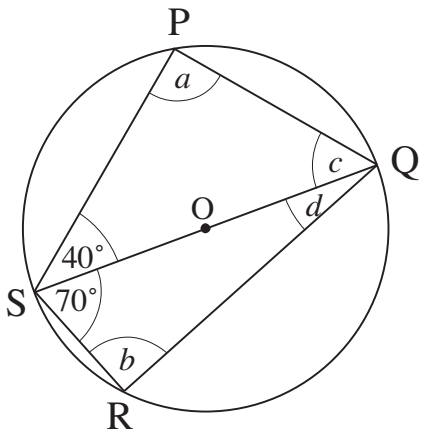


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Data Sheet H2.4 *Angles and Circles: Example 1*

Example 1

Find the angles marked with letters in the diagram, if O is the centre of the circle.



Solution

As both triangles are in semi-circles, angles *a* and *b* must each be

$\square^{o}$

Triangle PQS: $\square + \square + c = 180°$

$c = 180° - \square^{o}$

$c = \square^{o}$

Triangle QRS: $\square + \square + d = 180°$

$d = 180° - \square^{o}$

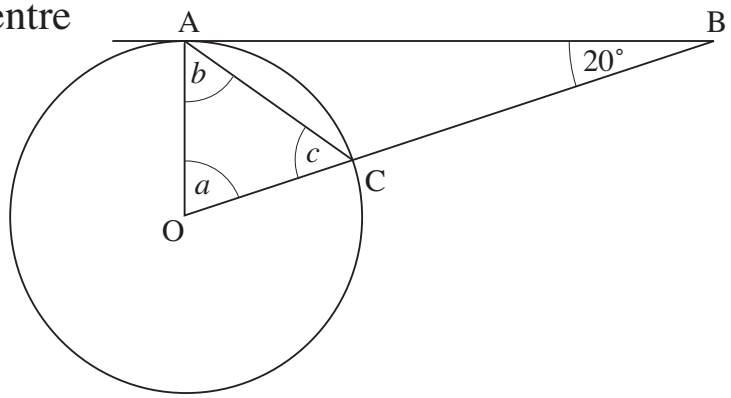
$d = \square^{o}$

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Data Sheet H2.5 *Angles and Circles: Example 2*

Example 2

Find the angles a, b and c, if AB is a tangent and O is the centre of the circle.



Solution

In triangle OAB, OA is a radius and AB a tangent,

so the angle between them = ☐°

Hence ☐° + ☐° + a = 180°

a = 180° − ☐°

a = ☐°

In triangle OAC, OA and OC are both radii of the circle.

Hence OAC is an ☐ triangle, and ☐ = ☐

2 × ☐ + ☐° = 180°

2 × ☐ = ☐°

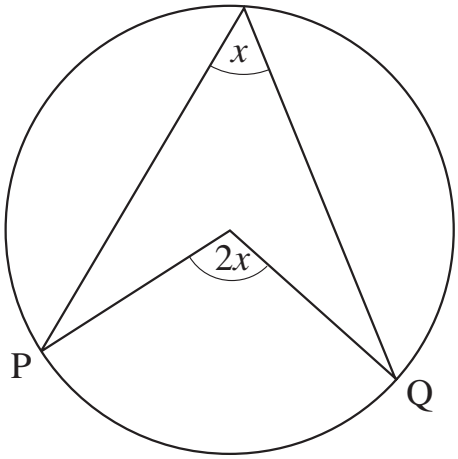
☐ = ☐°

and ☐ = ☐°

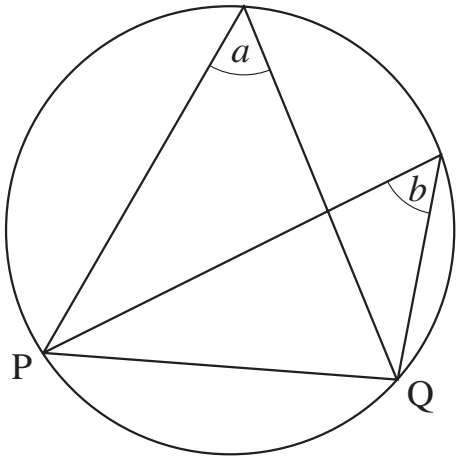
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Data Sheet H2.6 *Angles and Circles: More Results*

The angle subtended by an arc, PQ, at the centre is twice the angle subtended on the circumference.



Angles subtended at the circumference by a chord (on the same side of the chord) are equal; that is, in the diagram $a = b$.

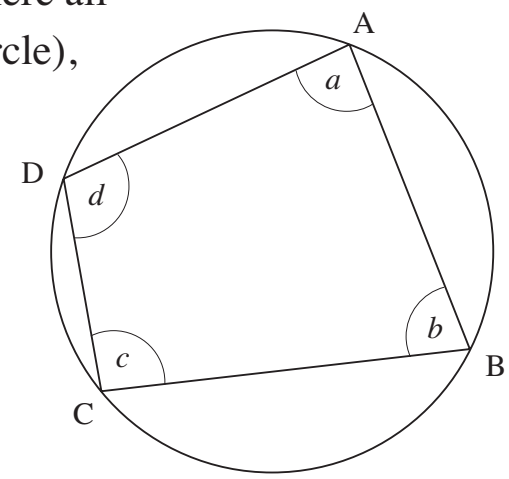


In *cyclic quadrilaterals* (quadrilaterals where all 4 vertices lie on the circumference of a circle), opposite angles sum to 180°; that is

$$a + c = 180°$$

and

$$b + d = 180°$$



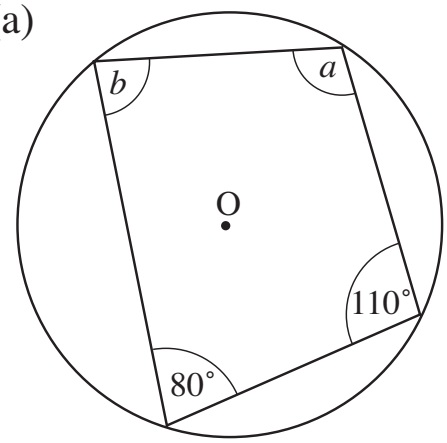
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Data Sheet H2.7 *Angles and Circles: More Examples*

Example

Find the angles marked in the diagrams. O is the centre of the circle.

(a)



Solution

Opposite angles in a cyclic quadrilateral add up to $\square^{o}$.

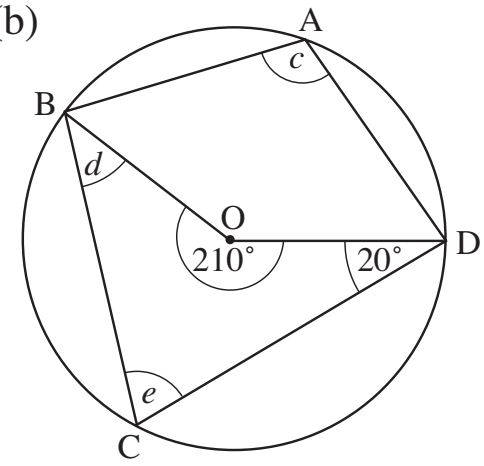
So $a + \square^{o} = \square^{o}$

$\Rightarrow a = \square^{o}$

and $b + \square^{o} = \square^{o}$

$\Rightarrow b = \square^{o}$

(b)



Solution

Consider arc BD. The angle subtended at O

$= \square \times c$

So $\square \times c = \square^{o}$

$\Rightarrow e = \square^{o}$

Also, $c + e = \square^{o}$

$\square + e = \square^{o}$

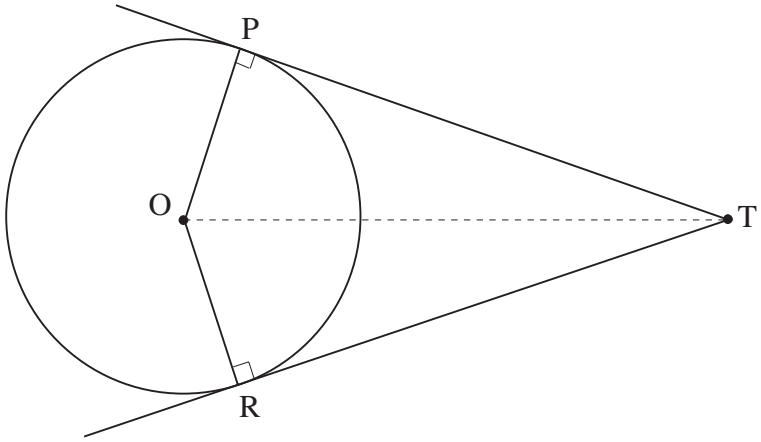
$e = \square^{o} - \square^{o}$

$= \square^{o}$

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Data Sheet H2.8 *Circles and Tangents: Results A*

1. If two tangents are drawn from a point T to a circle with centre O, and P and R are the points of contact of the tangents with the circle, then, using symmetry,

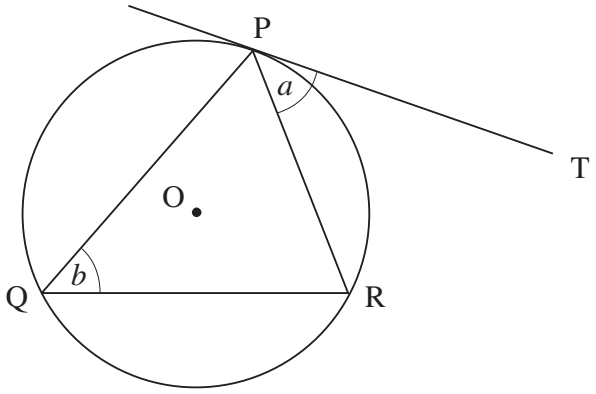


(a) PT = RT

(b) triangles TPO and TRO are congruent.

2. The angle between a tangent and a chord equals an angle on the circumference subtended by the same chord;
e.g. $a = b$ in the diagram.

This is known as the *alternate segment theorem*

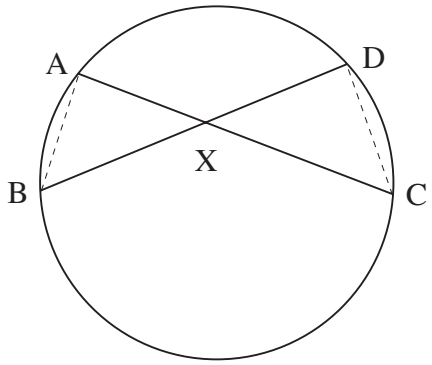


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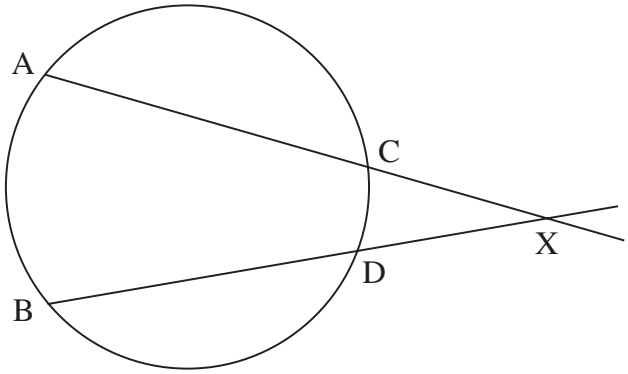
Data Sheet H2.9 *Circles and Tangents: Results B*

3. For any two intersecting chords, as shown,

$$AX \times CX = BX \times DX$$



4. This result will still be true even when the chords intersect *outside* the circle, as illustrated opposite.



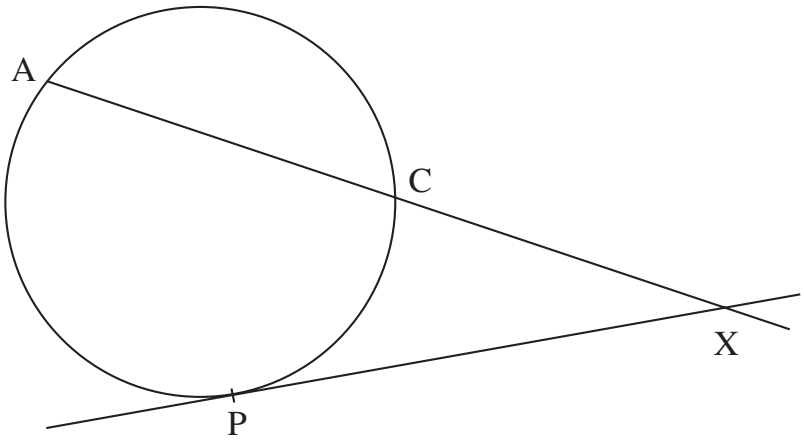
5. When the chord BD becomes a tangent,

and B and D coincide at P, the point of tangency, then

$$AX \times CX = PX \times PX$$

or

$$AX \times CX = PX^2$$

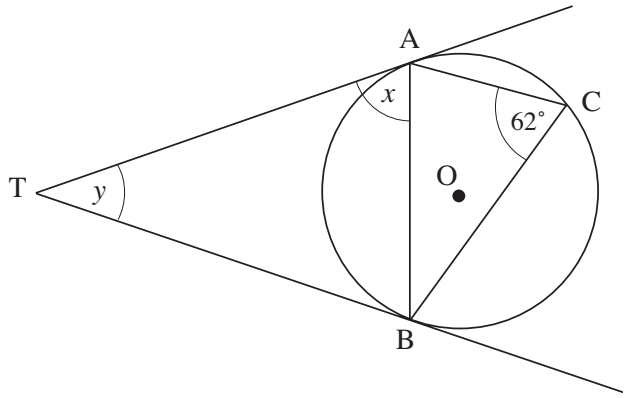


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Data Sheet H2.10 *Circles and Tangents: Example 1*

Example 1

Find the angles x and y in the diagram.



Solution

From the alternate angle segment theorem, $x =$ ☐ °.

Since TA and TB are equal in length, ΔTAB is ☐ and

angle ABT = ☐ °.

Hence $y +$ ☐ ° + ☐ ° = 180°

i.e. $y =$ ☐ ° − ☐ °

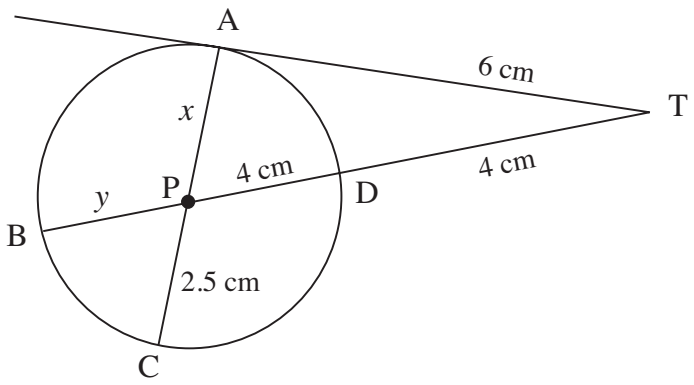
= ☐ °

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Data Sheet H2.11 *Circles and Tangents: Example 2*

Example 2

Find the unknown lengths in the diagram.



Solution

Since AT is a tangent,

$$AT^2 = BT \times \square$$

So $$BT = \frac{\square}{\square} = \square \text{ cm}$$

Thus $$y + \square = \square \Rightarrow y = \square \text{ cm}$$

Since AC and BD are intersecting chords,

$$AP \times PC = \square \times \square$$

$$\square \times x = \square \times \square$$

$$x = \frac{\square}{\square}$$

$$= \square \text{ cm}$$

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