H2 *Angles, Circles and Tangents* Introduction

Learning objectives

This unit continues the angle geometry theme, bringing in the important topic of *bearings* and also using circles and tangents. After completing Unit H2 you should

- be able to identify and use bearings in practical problems
- ✻ understand and be able to use angle geometry results for circles, chords, diameters and tangents
- ✻ appreciate and use the subtended angle results in a circle and the concept of a cyclic quadrilateral
- ✻ understand and be confident in using the alternate segment theorem and the length formula for intersecting chords.

Introduction

Angle geometry is introduced in Unit H1 and is extended in this unit to working with *circles* and *tangents*. Another aspect is introduced - the concept of *compass bearings* for specifying direction.

This topic has its own history.

It is thought that the Chinese knew about the 'magical' magnetic properties of the iron mineral ore called lodestone (or loadstone) as early as the 4th century BC.

At that time they had no idea that while the iron ore was still hot volcanic magma, its atoms had aligned with the earth's magnetic field and, as it cooled back to rock, it became magnetised.

The ancient Greeks also knew about the weak attraction lodestone had to tin but the compass's first actual use to get bearings at sea, is placed closer to around 300 AD. Chinese sailors were reported to be using it around 1100 AD. Arabian merchants had started using it by 1220 AD and the Scandinavian Vikings by 1250 AD. Marco Polo brought Europe's first compass back from the Chinese city of Cathay at the close of the 1200s.

In the early days of the compass, it was only a small piece of rock ore, strapped to a small floating piece of wood. It could even be hung by a thread.

The early navigators found the magnetic compass useful because it was very easy to use, it made steering a ship much more accurate and it could even be used when the skies were cloudy, when the more accurate method of navigating by the stars was not possible.

In all the past centuries of adventure, when men of courage sailed strange and dangerous seas and blazed trails across wild unsettled continents, the direction of north was their one and only fixed point on the globe.

In the earliest charts, north is indicated by a spearhead above the letter T (for tramontana, a northerly wind). This 3-pointed symbol came to stand for the north. It was first seen on Portuguese maps around the time of Columbus (15th century).



Spearhead (or fleur-de-lys) figure indicating north

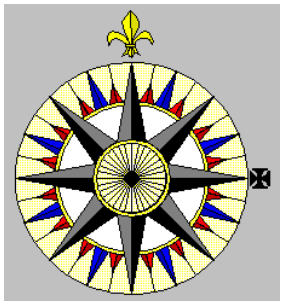
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With slight changes, the symbol was adopted as the emblem of the Scouting movement in the early 20th century.

Scouting emblem



14th century compass rose

In the 14th century, the east side of the rose was indicated with a cross, indicating the direction to Paradise (thought to be in the east), or at least to where Christ was born (in the Levant, Eastern Mediterranean).

Before the magnetic compass was discovered, early map makers would draw a small 16-pointed circle on the map indicating the 4 cardinal points and the intervening points. The 32 points of the compass rose come from the directions of the eight major winds, the eight half-winds and the sixteen quarter-winds.

The points (or bearings) shown are

N, NNE, NE, ENE, E, ESE, SE, SSE, S, SSW, SW, WSW, W, WNW, NW, NNW, N

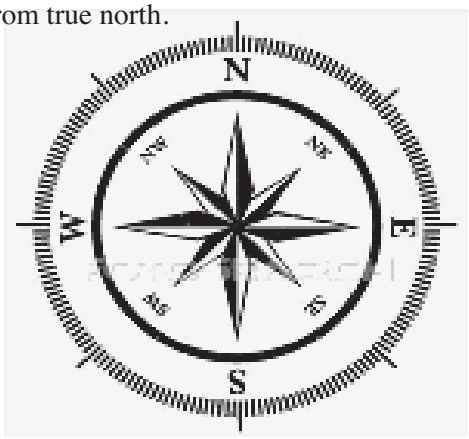
N (north), S (south) , E (east) and W (west) are the 4 *cardinal points* (or principal points).

When the magnetic compass came into use, it was just set on top of the wind rose pattern in order to help face the sea chart in the proper direction.

There is no absolute standard for drafting a compass rose, and each school of cartographers seems to have developed their own.

The colours on the figure are supposedly the result of the need for graphic clarity rather than a mere cartographical whim. On a rolling ship at night by the light of a flickering lamp, these figures had to be clearly visible. Therefore the eight principle points of the compass are usually shown on the compass rose in black which stands out easily. Against this background, the points representing the half-winds are typically coloured in blue or green and since the quarter-wind points are the smallest, they are usually coloured red.

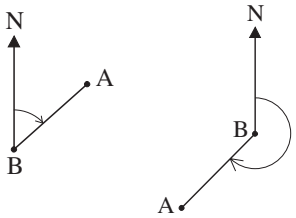
This was the extent of the accuracy of an old Mariner's Compass and, by today's standards, it was not at all accurate. However, as global spherical mathematics improved, it became more customary to give bearings in units of 'degrees' from geographic north. In the 1920s it became an accepted worldwide practice to indicate direction or *bearing* by a single 3-digit number (0 to 360) representing degrees of a circle as measured clockwise from true north.



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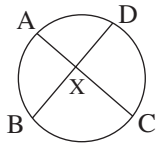
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Key points and principles



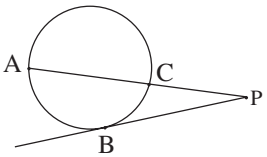
- In finding bearings, say A from B, you must find the angle, measured *clockwise*, made by the N direction with the line from B to A.
- The angle subtended at the circumference of a circle by a diameter is a right angle.
- The angles subtended at the circumference on the same side of a circle by a chord are equal.
- The angle subtended at the centre of a circle by an arc is twice the angle subtended at a point on the circumference (alternate segment theorem).
- Opposite angles of a cyclic quadrilateral sum to $180°$.
- The angle subtended between a tangent and a chord equals any angle on the circumference subtended by the chord.
- For the circle,

  $AX \times CX = BX \times DX$



- For the circle,

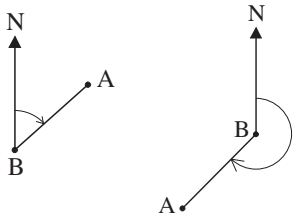
  $BP^2 = AP \times CP$



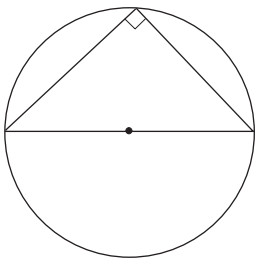
Facts to remember

- *Bearings* are of the form of angles, expressed as three-digit numbers and are measured from the N, in a *clockwise* direction.

  In finding bearings, say A from B, you must find the angle, (measured clockwise) made by the N direction with the line from B to A.



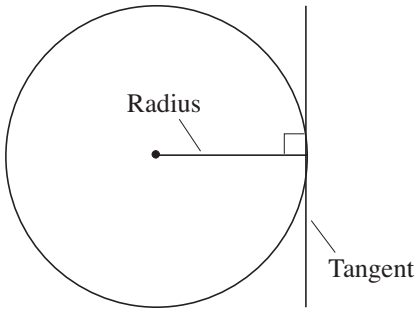
- When a triangle is drawn in a semi-circle as shown, the angle at the perimeter is always a right angle.



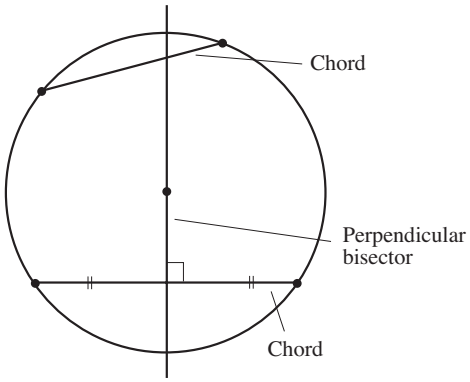
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- A *tangent* is a line that touches only one point on the circumference of a circle. The point is known as the *point of tangency*.

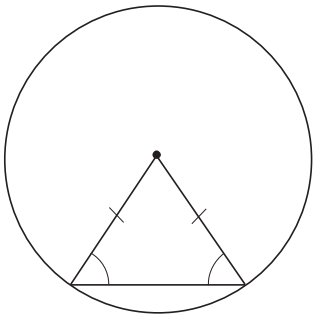
  A tangent is always perpendicular to the radius of the circle.



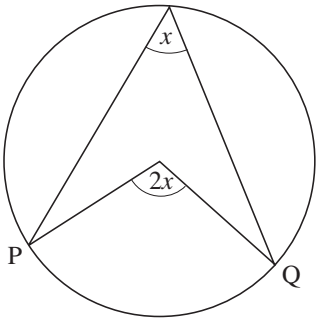
- A *chord* is a line joining any two points on the circle.
- The *perpendicular bisector* is a second line that divides the chord in half and is at right angles to it.
- The perpendicular bisector of a chord always passes through the centre of the circle.



- When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed, so the two angles marked are equal.

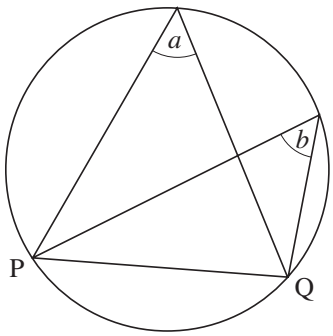


- The angle subtended by an arc, PQ, at the centre is twice the angle subtended at the circumference.



- Angles subtended at the circumference by a chord (on the same side of the chord) are equal;

  that is, in the diagram $a = b$.

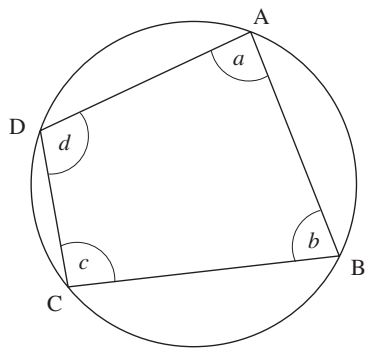


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- In *cyclic quadrilaterals* (quadrilaterals where all 4 vertices lie on the circumference of a circle), opposite angles sum to 180°; that is

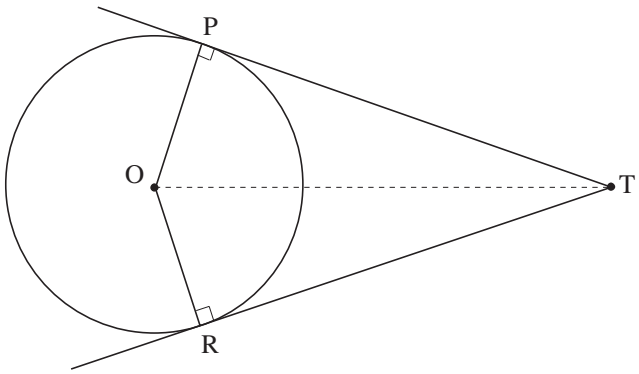
$$a + c = 180°$$
and $$b + d = 180°$$



- If two tangents are drawn from a point T to a circle with centre O, and P and R are the points of tangency, then, using symmetry

$$PT = RT$$

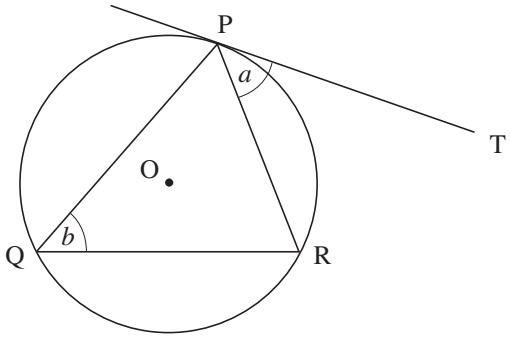
and triangles TPO and TRO are congruent.



- The angle between a tangent and a chord equals an angle at the circumference subtended by the same chord; e.g. $a = b$ in the diagram.

  This is known as the

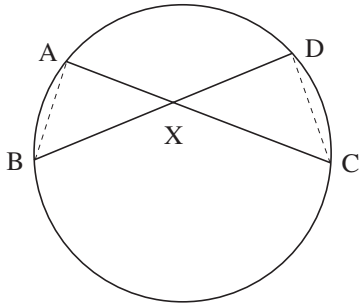
  alternate segment theorem.



- For any two intersecting chords, as shown,

$$AX \times CX = BX \times DX$$

The proof is based on similar triangles.

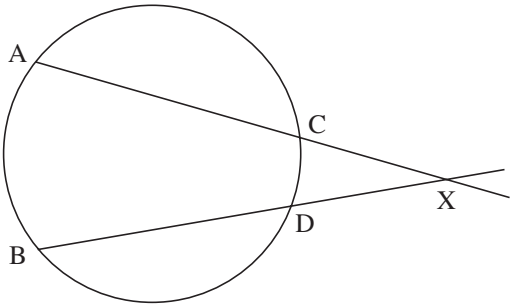


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This result will still be true even when the chords intersect *outside* the circle, as illustrated opposite.

How can this be proved?



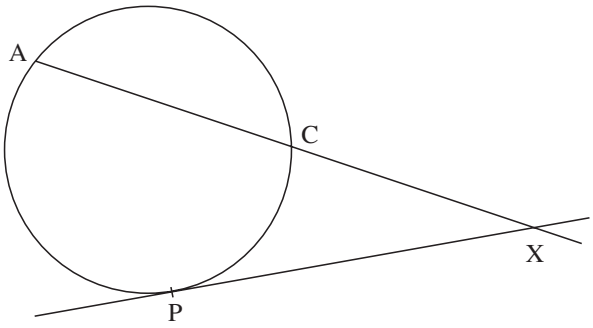
When the chord BD becomes a tangent,

and B and D coincide at the point P, then

$$AX \times CX = PX \times PX$$

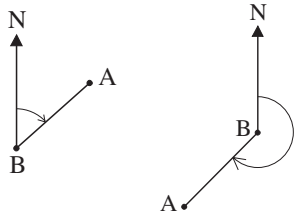
or

$$AX \times CX = PX^2$$

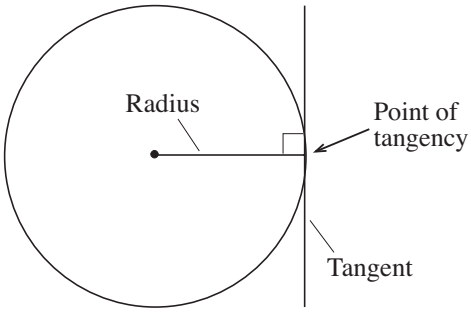


Glossary of terms

- A *bearing* gives the direction or position of something, or the direction of movement, relative to a fixed point. Bearings are of the form of angles, expressed in degrees as three-digit numbers; they are measured from north in a clockwise direction.
  For example, 060°, 210°.



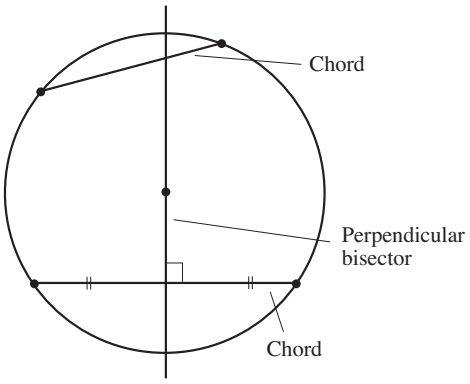
- A *tangent* is a line that touches only one point on the circumference of a circle.
  A tangent is always perpendicular to the radius of the circle.
- The *point of tangency* is the point where a tangent touches the circumference of a circle.



- A *chord* is a line joining any two points on the circle.

  The *perpendicular bisector* is a second line that divides the first line in half and is at right angles to it.

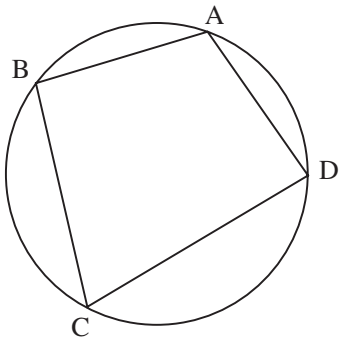
  The perpendicular bisector of a chord always passes through the centre of the circle.



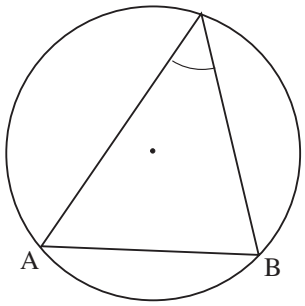
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- A *cyclic quadrilateral* is a quadrilateral with all four vertices on the circumference of a circle.



- An a*ngle subtended by a chord* in a circle is shown in the diagram opposite.



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