UNIT I1 *Pythagoras' Theorem and Trigonometric Ratios* Activities

Activities

I1.1 Spirals
I1.2 Clinometers
I1.3 Posting Parcels
I1.4 Exact Values
I1.5 Trigonometric Puzzle

Notes and Solutions (2 pages)

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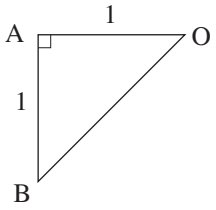
ACTIVITY I1.1 *Spirals*

Spirals of various kinds are found throughout nature in both plants and animals. Examples include *Snail's shell* *Sheep's horns* *Seashells* *Fossils*.

We will show here how to construct spirals and how they can be used.

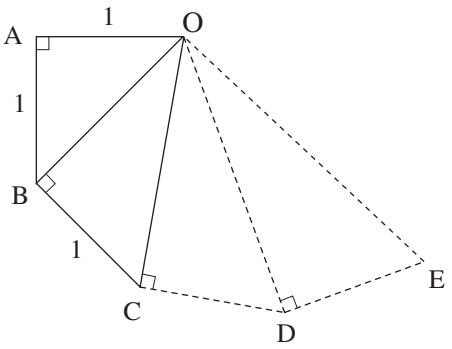
Geometric Construction

Step 1 Construct an isosceles triangle with two sides, OA and AB, of unit length.



Step 2 Add another line, BC, of unit length, perpendicular to the longest side, OB, of the triangle and complete the next triangle by drawing OC.

Step 3 Continue in this way, each time adding a unit length.

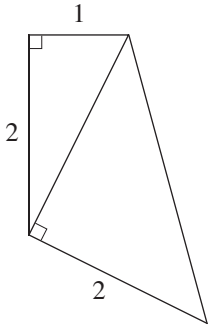


1. Draw the spiral in this way with 15 linked sides. What is the length of

   (a) OB (b) OC (c) OD? (Use Pythagoras' Theorem.)

2. From your construction, estimate the value of $\sqrt{14}$. Check your accuracy using a calculator.

3. Repeat the construction, starting with a right angled triangle as shown, adding on sides of length 2 units.
   How many sides do you need to construct in order to estimate $\sqrt{33}$?

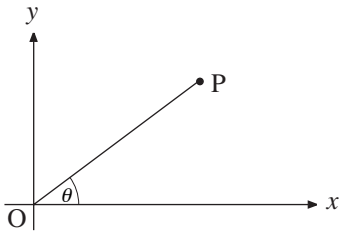


Extension

As it consists of straight lines, the construction used above produces only an *approximation* to a true spiral. We can, though, obtain a true spiral using the equation

$$r = \left(\frac{\theta}{360}\right)$$

Here r is the distance of the point P from the origin, where OP makes an angle θ with the x axis. Plot the points which correspond to $\theta = 0°, 45°, 90°, \ldots\ldots, 360°$, and join up the points with a smooth curve.
Continue the curve by plotting points corresponding to $\theta = 405°, 450°, \ldots\ldots,$ etc.



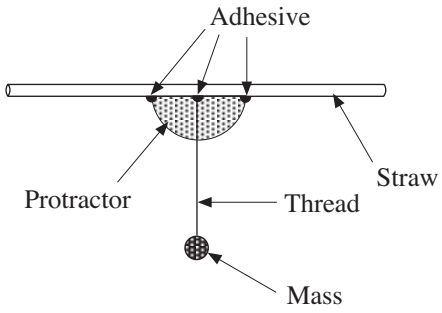
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ACTIVITY I1.2 *Clinometers*

A *clinometer* is an instrument for measuring the angle of an incline (slope).

To make your clinometer, you need a drinking straw, a protractor, some adhesive, a small mass and a piece of thread.

Use adhesive to attach the straw to the protractor. The thread with a mass at its end will serve as a plumb line.

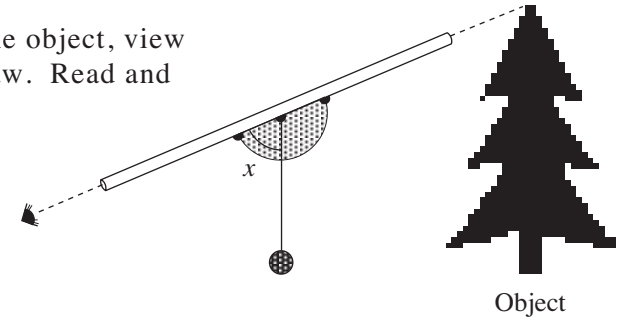


You can now use your clinometer to find the height of an object such as a tree or a building.

Follow this method.

1. Choose your object.

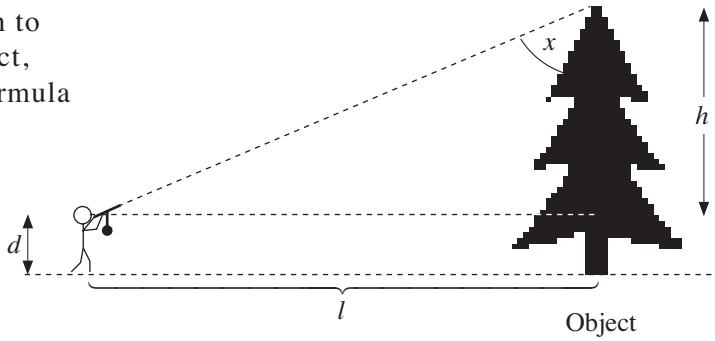
2. Standing some distance away from the object, view the top of it through the drinking straw. Read and record the value of angle x.



3. Measure and record your distance, l, from the object.

4. Use the readings you have taken to calculate the height of the object, above your eyeline, from the formula

$$\tan x = \frac{l}{h} \Rightarrow h = \frac{l}{\tan x}$$



5. The height, H, of the object is now given by

$$H = \frac{l}{\tan x} + d$$

when d is the height of your eyes from the ground.

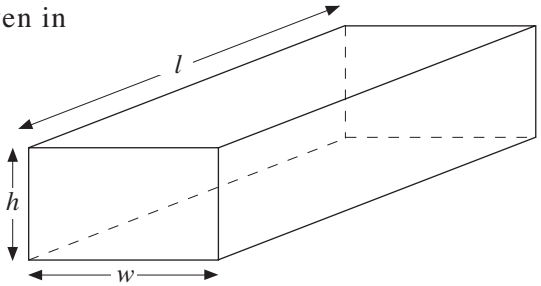
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ACTIVITY I1.3 *Posting Parcels*

The UK's *Royal Mail Data Post International* parcel service accepts parcels up to a maximum size as given in the rules below.

Rule 1 *Length* (l) + *height* (h) + *width* (w) must not exceed 900 mm.

Rule 2 None of the *length, height, width* must exceed 600 mm.

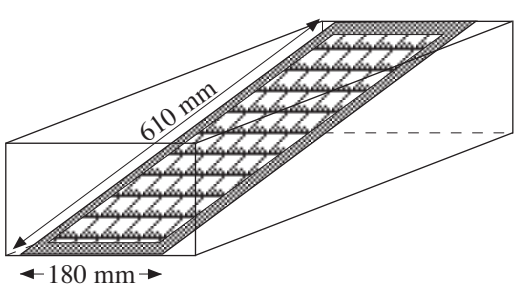


1. Which of the following parcels would be accepted for this service?

   (a) l = 620 mm, h = 120 mm, w = 150 mm

   (b) l = 500 mm, h = 350 mm, w = 150 mm

   (c) l = 550 mm, h = 100 mm, w = 150 mm

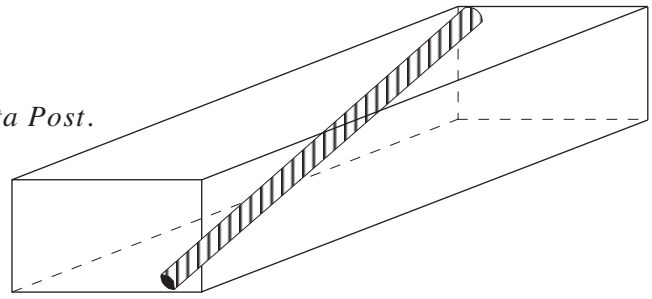
2. 

   A picture with frame, 610 mm by 180 mm, is to be placed diagonally in a rectangular box as shown.

   Find suitable dimensions for the box so that it would be accepted for the *Data Post* service.

3. A long, thin tube is to be sent by *Data Post*.

   What is the largest possible length that can be sent?



Extension

What are the dimensions of the rectangular box of *maximum volume* that can be sent through the *Data Post* service?

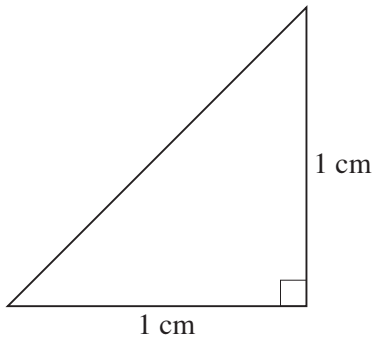
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ACTIVITY I1.4 *Exact Values*

For some angles we can determine exact values, using fractions and square roots instead of just decimal approximations.

For example, the *exact* value of $\cos 30° = \frac{\sqrt{3}}{2}$. These exact values do not contain any rounding errors and can be very useful.

1. The diagram shows an isosceles triangle.

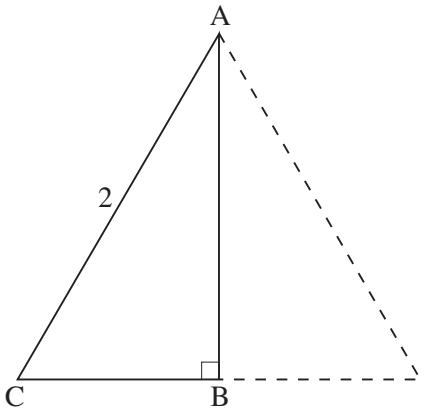


   (a) State the sizes of the angles in the triangle.

   (b) Calculate the length of the hypotenuse.

   (c) Determine exact values for $\sin 45°$, $\cos 45°$ and $\tan 45°$ using this triangle.

2. The diagram shows an equilateral triangle that has been cut in half to form the right-angled triangle ABC.



   (a) What is the size of each of the angles in this triangle?

   (b) What are the lengths of BC and AB ?

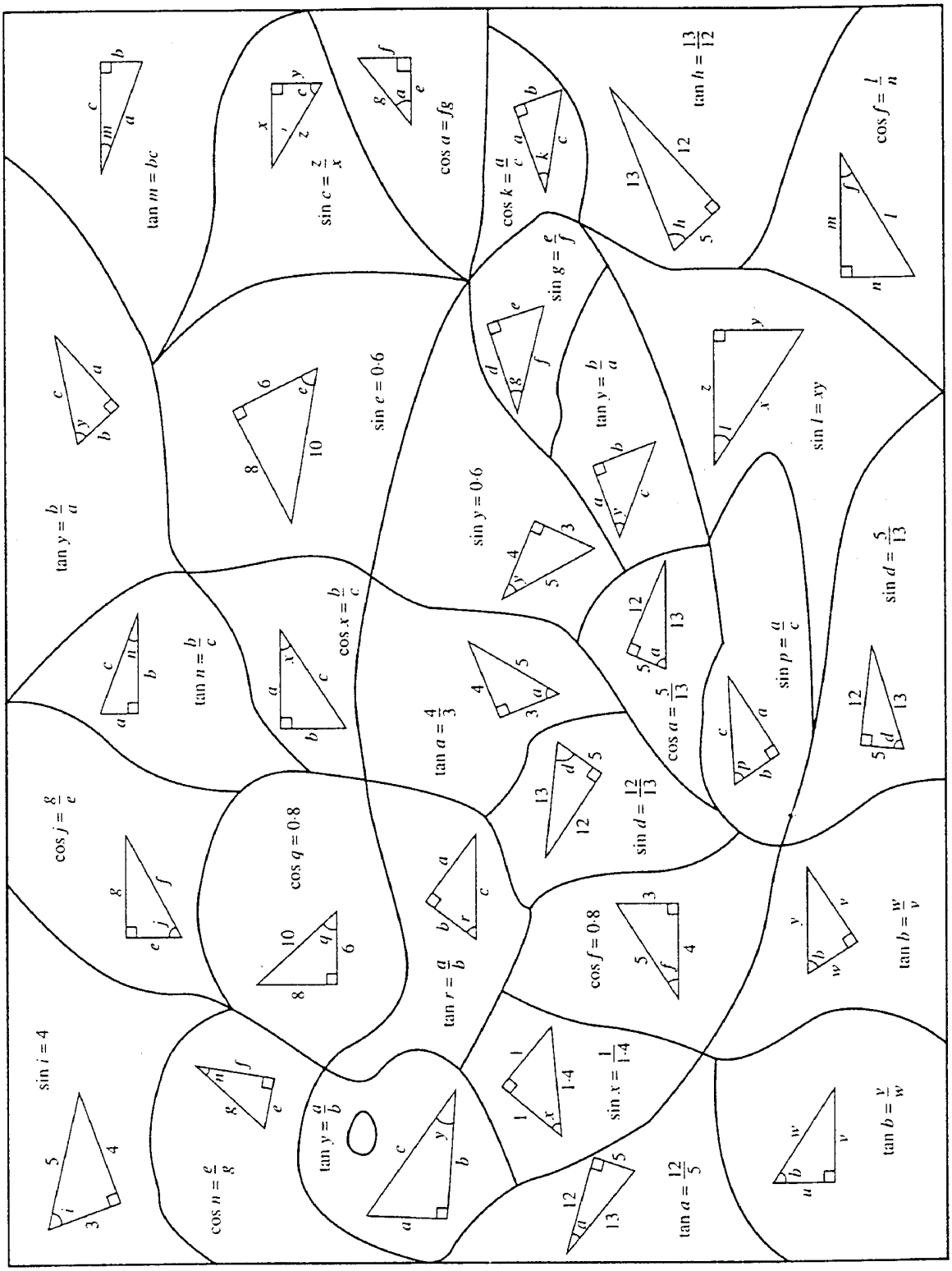
   (c) Determine exact values for $\sin 60°$, $\cos 60°$ and $\tan 60°$.

   (d) Determine exact values for $\sin 30°$, $\cos 30°$ and $\tan 30°$.

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ACTIVITY I1.5 *Trigonometric Puzzle*

Shade all the regions which display *correct* trigonometric ratios to find the hidden animal.



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ACTIVITIES I1.1- I1.4 *Notes and Solutions*

Notes and solutions given only where appropriate.

I1.1 1. (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{4}$

2. 3.74 (2 d. p.)

3. 8 sides needed

I1.3 1. (a) No $(l > 600)$ (b) No $(l + h + w > 900)$ (c) Yes

2. Many possibilities

3. About 671 mm

I1.4 1. (a) $45\,°,\ 45\,°,\ 90\,°$

(b) $\sqrt{2}$ cm

(c) $\sin 45\,° = \frac{1}{\sqrt{2}},\quad \cos 45\,° = \frac{1}{\sqrt{2}},\quad \tan 45\,° = 1$

2. (a) $30\,°,\ 60\,°,\ 90\,°$

(b) $\mathrm{BC} = 1,\quad \mathrm{AB} = \sqrt{3}$

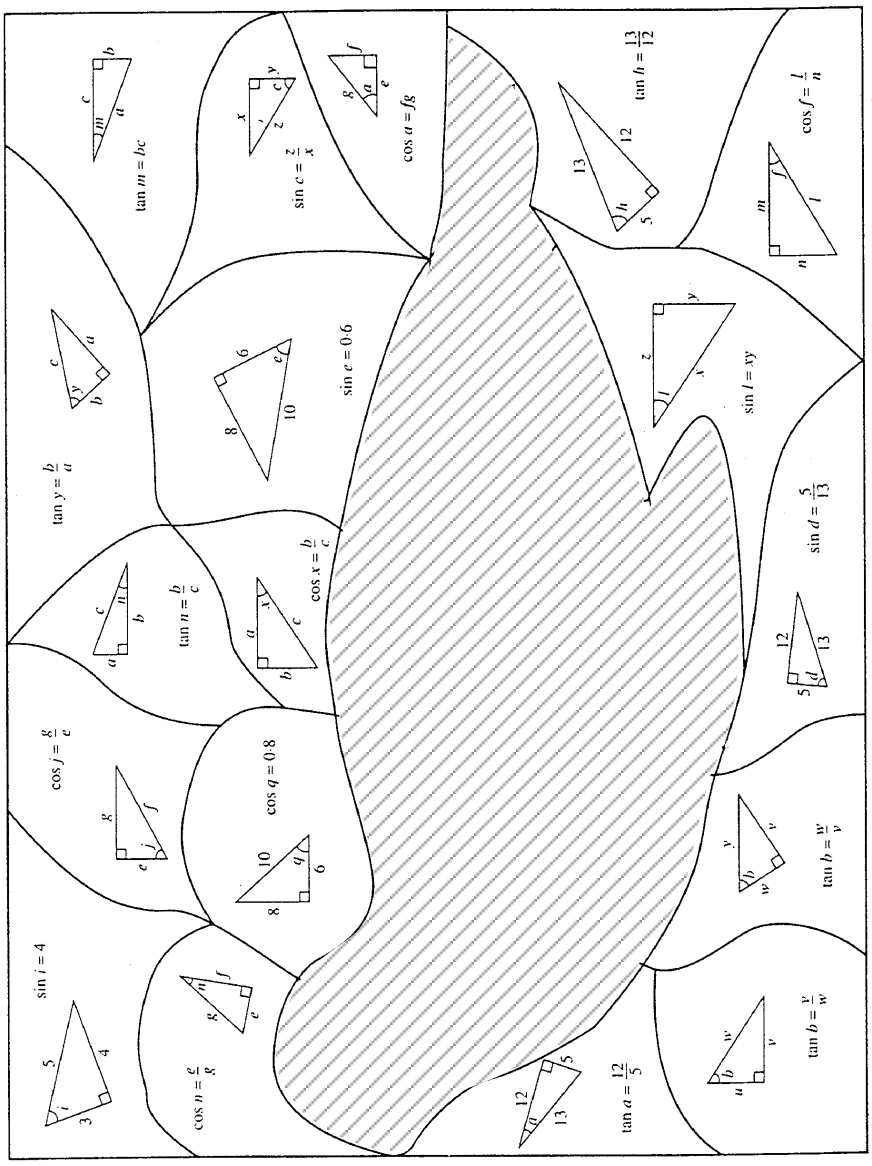
(c) $\sin 60\,° = \frac{\sqrt{3}}{2},\quad \cos 60\,° = \frac{1}{2},\quad \tan 60\,° = \sqrt{3}$

(d) $\sin 30\,° = \frac{1}{2},\quad \cos 30\,° = \frac{\sqrt{3}}{2},\quad \tan 30\,° = \frac{1}{\sqrt{3}}$

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ACTIVITY I1.5 *Notes and Solutions*

I1.5 Bird



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