UNIT I2 *Trigonometric Problems* Activities

Activities

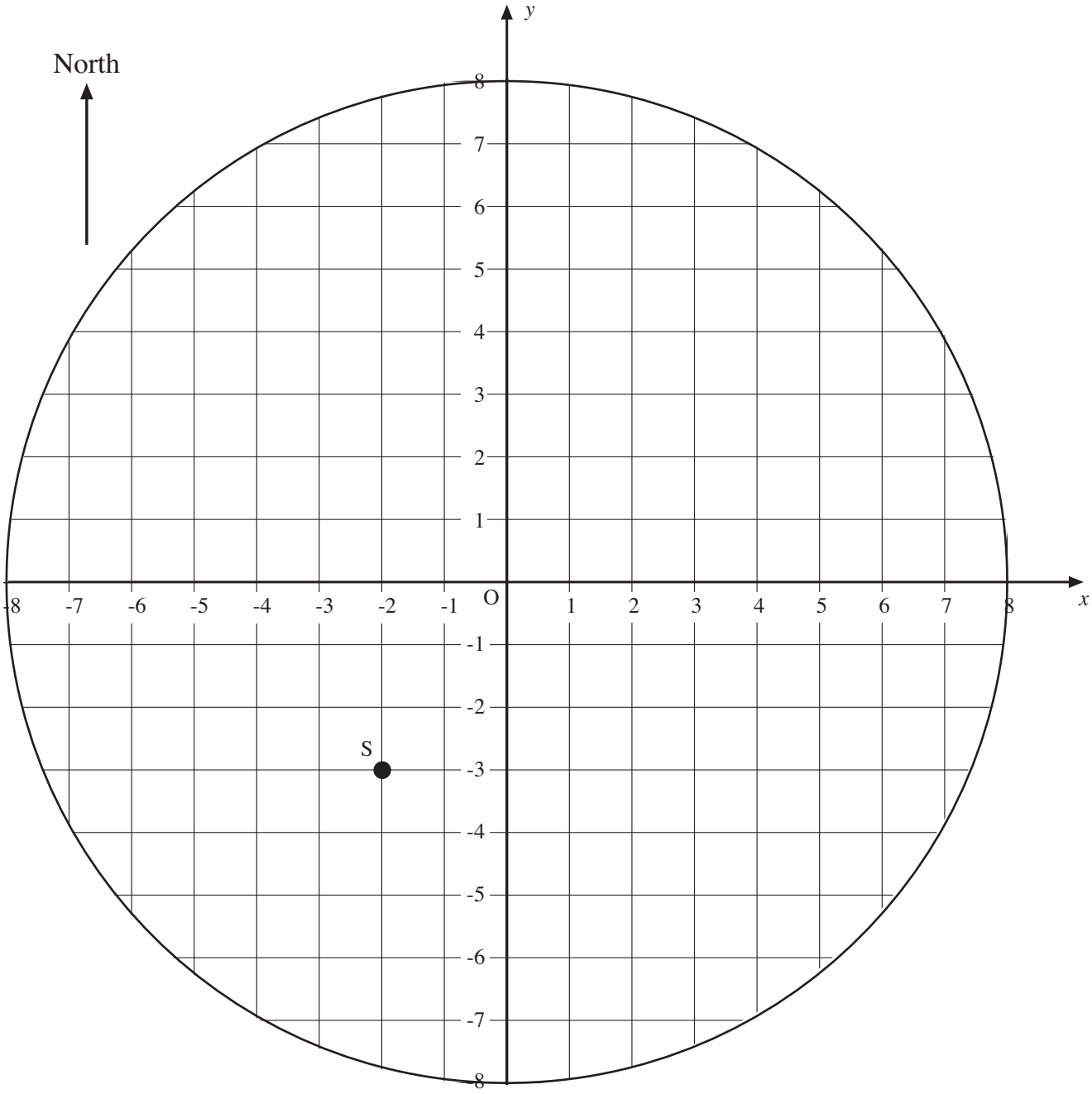
I2.1 Radar (2 pages)
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ACTIVITY I2.1 *Radar*

A ship is anchored at sea. The following diagram represents the circular screen of the ship's radar display. It is divided into squares. Each side of a square represents **10 km**. The centre of the screen, O, represents the ship's own position. Another ship, S, is anchored at (–2, –3). Its position is indicated by a blip on the screen. (In the diagram, we use a dot to represent the blip.)



(continued)

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ACTIVITY I2.1 (continued) *Radar*

1. What is the distance between the two ships, correct to the nearest km?

2. At 0600, another blip appears at (5, 6) with respect to O, travelling on a bearing of $200°$ at an estimated speed of 40 km/h.

   (a) Use a dot to indicate the position of this unidentified vessel on the screen. What is its distance from the ship at O, correct to the nearest km?

   (b) Draw a line clearly on the screen to show the course of navigation taken by this unidentified vessel.

   (c) If this unidentified vessel continues with the same course, what will be the shortest distance between it and the ship at O?
   Give your answer correct to the nearest km.

   (d) How long will it be from the time the blip first appears to the time the unidentified vessel moves out of the radar display? How far will it then be from O?

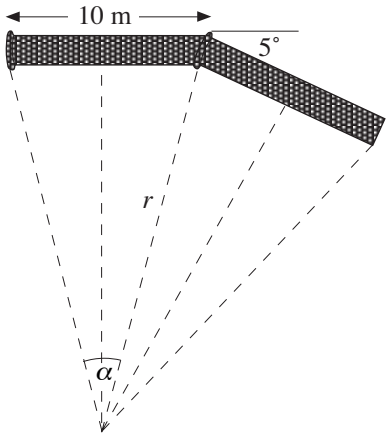
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ACTIVITY I2.2 *Interlocking Pipes*

Pipes which are used to drain away surface rainwater from roads are designed with a *lip* in such a way that 10 metre pipes can have a change in angle of at most $5°$.

The Highways Department wants to lay these pipes along a road which has a curvature of radius 150 metres. Is this possible?

You can answer this problem by finding the minimum radius that can be obtained by using the maximum angle change of $5°$. The situation is illustrated below.



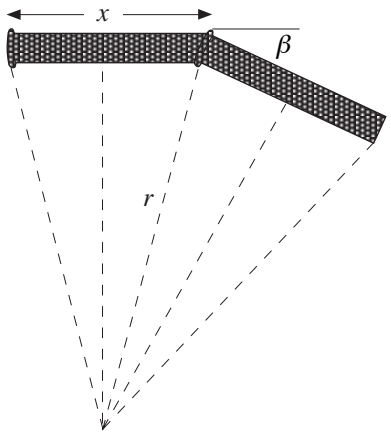
1. What is the value of the angle α as shown in the diagram?

2. Show that $r = \dfrac{5}{\sin \frac{1}{2}\alpha}$.
   Hence find the minimum radius possible.

3. Can a curvature of 150 metres be achieved? If so, what should be the angle of change between pipes?

Although you have solved a particular problem, it would be very useful for the highway engineer to have a way of finding a *general* expression for the minimum radius possible, say r metres, for pipes of a given length, say x metres, which has a maximum angle change of $\beta°$, as shown below.

4. Find the relationship between r, x and β.



r \ x	2	5	10	20
50	...	...	11.48	...
75	...	...	...	...
100	...	...	...	...
125	...	...	...	...
150	...	...	3.82	...

5. Use this relationship to complete the ready reckoner table as shown above for the required angle β, given r and x.

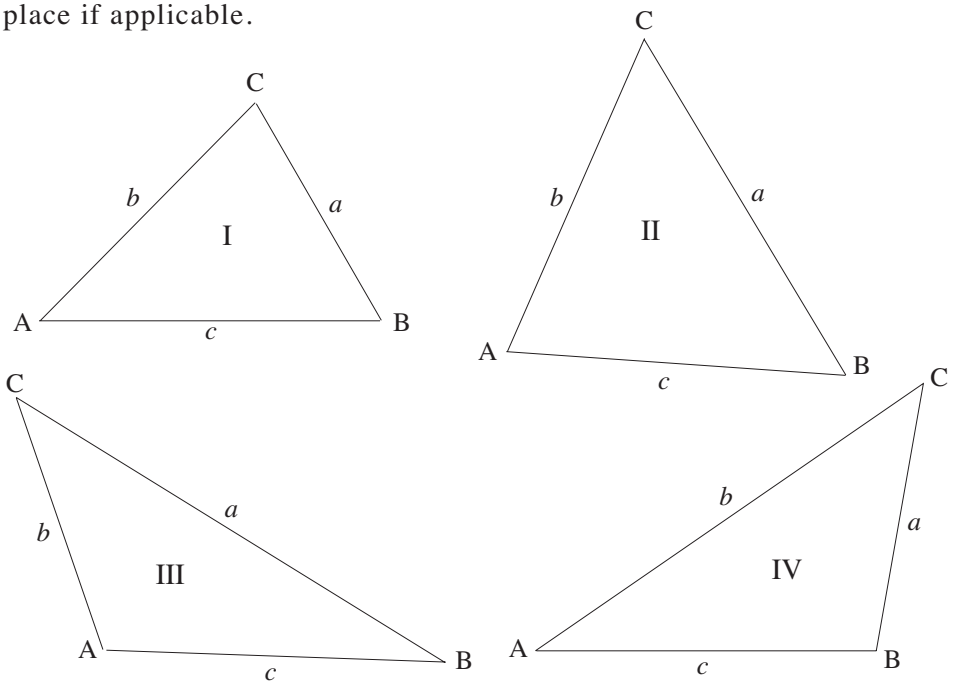
6. If pipes are available only in sizes of 2, 5, 10 and 20 metres, find the maximum size to produce a radius of curvature of

   (a) 125 metres (b) 75 metres.
   (Assume that $5°$ is the maximum change in angle possible.)

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ACTIVITY I2.3 *Sine Rule*

Measure the sizes of all three angles and the lengths of all three sides of each of the following triangles. Complete the table that follows. Give your answers correct to one decimal place if applicable.



Triangle	∠A	∠B	∠C	a	b	c	$\frac{a}{\sin A}$	$\frac{b}{\sin B}$	$\frac{c}{\sin C}$
I									
II									
III									
IV									

What can you conjecture from these results?

Extension

Draw a circle of a certain diameter and draw any triangle ABC with its vertices on the circumference of the circle. Measure the sides and angles of the triangle, and compare the values of

$$\frac{a}{\sin A}, \quad \frac{b}{\sin B}, \quad \frac{c}{\sin C}$$

to the diameter. Repeat with a circle of different diameter.

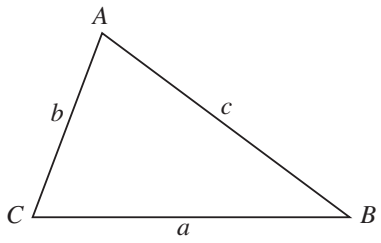
What can you conjecture?

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ACTIVITY I2.4 *Heron's Formula*

The area, A, of a triangle with sides of lengths a, b and c is given by Heron's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$



where $s = \dfrac{a+b+c}{2}$.

There are three alternative versions of this formula; these are the focus of this activity.

Theorem 1

Heron's formula can be written in terms of a, b and c as

$$A = \frac{1}{4}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}$$

Proof Substitute $s = \dfrac{a+b+c}{2}$ in Heron's formula to show this result.

Theorem 2

Heron's formula can be written as

$$A = \frac{1}{4}\sqrt{2\left(a^2b^2 + b^2c^2 + c^2a^2\right) - \left(a^4 + b^4 + c^4\right)}$$

Proof Multiply out the expression in Theorem 1 to show this.

Theorem 3

Heron's formula can be written as

$$A = \frac{1}{4}\sqrt{4a^2c^2 - \left(a^2 + c^2 - b^2\right)^2}$$

Proof Multiply out the expression inside the square root to show that it is the same as Theorem 2.

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ACTIVITIES I2.1 - I2.4 *Notes and Solutions*

Notes and solutions given only where appropriate.

I2.1 1. 36 km

2. (a) 78 km (c) 26 km (d) almost 4 hours ; 80 km

I2.2 1. 5°

2. $r \approx 114.6\,\text{m}$

3. Yes ; 3.82°

4. $r = \dfrac{x}{2\sin\left(\dfrac{\beta}{2}\right)}$

6. (a) 10 m (b) 5 m

I2.3 The result should confirm the sine rule; the extension relates to the result that $\dfrac{a}{\sin a} = \dfrac{b}{\sin b} = \dfrac{c}{\sin c} = 2d$ when d is the diameter of the circle.

I2.4 **Theorem 1** *Proof*

$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ when } s = \frac{a+b+c}{2}$$

$$s - a = \frac{a+b+c}{2} - a = \frac{b+c-a}{2}, \text{ etc, giving}$$

$$s = \sqrt{\frac{(a+b+c)}{2}\frac{(b+c-a)}{2}\frac{(c+a-b)}{2}\frac{(a+b-c)}{2}}$$

$$= \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)} \text{ as required.}$$

Theorem 2 *Proof*

Expanding $(a+b+c)(b+c-a) = (b+c)^2 - a^2 = b^2 + c^2 - a^2 + 2bc$

$$(c+a-b)(a+b-c) = (a+c-b)(a-(c-b)) = a^2 - (c-b)^2$$
$$= a^2 - b^2 - c^2 + 2bc$$

So $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$

$$= (b^2 + c^2 - a^2 + 2bc)(a^2 - b^2 - c^2 + 2bc)$$
$$= ((b+c)^2 - a^2)(a^2 - (b-c)^2)$$
$$= -a^4 - ((b+c)(b-c))^2 + a^2(b-c)^2 + a^2(b+c)^2$$
$$= -a^4 - (b^2 - c^2)^2 + a^2(b^2 - 2bc + c^2) + a^2(b^2 + 2bc + c^2)$$
$$= -a^4 - b^4 - c^4 + 2b^2c^2 + 2a^2b^2 + 2a^2c^2$$

Theorem 3 *Proof*

Expand the given formula, i.e. $4a^2c^2 - (a^2 + c^2 - b^2)^2$ to show it equates to Theorem 2.

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