I2 *Trigonometric Problems* Data Sheets

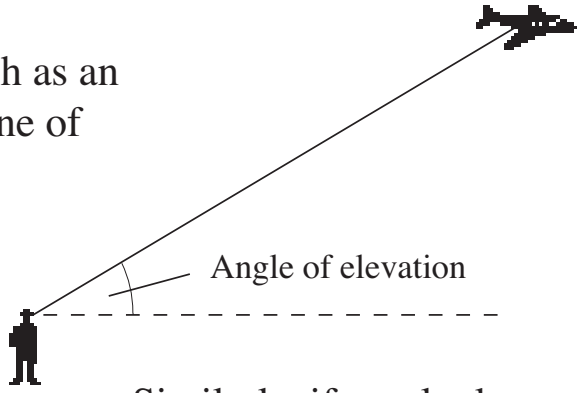
Data Sheets

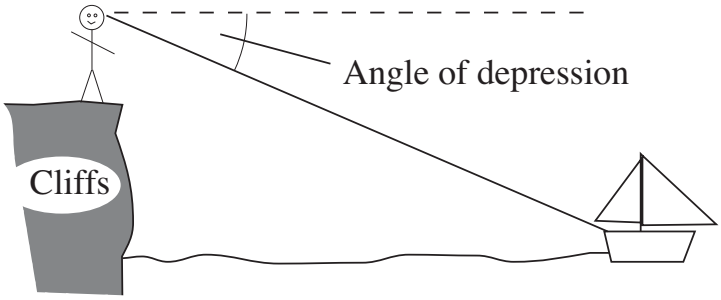
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Data Sheet I2.1 *Problems Using Trigonometry 1*

When you look *up* at something, such as an aeroplane, the angle between your line of sight and the horizontal is called the *angle of elevation*.





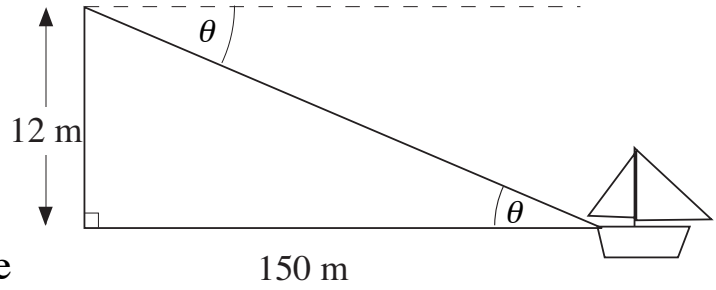
Similarly, if you look *down* at something, then the angle between your line of sight and the horizontal is called the *angle of depression*.

Example

A man looks out to sea from a cliff top at a height of 12 metres. He sees a boat that is 150 metres from the cliffs. What is the angle of depression?

Solution

The situation can be represented by the triangle shown in the diagram, where θ is the angle of depression.



$$\text{Using } \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{\square}{\square} = \square$$

$$\theta = \tan^{-1}\square$$

$$\text{So } \theta = \square^{\circ} \text{ to 1 decimal place}$$

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Data Sheet I2.2 *Problems Using Trigonometry 2*

Example A ladder is 3.5 metres long. It is placed against a vertical wall so that its foot is on horizontal ground and it makes an angle of $48°$ with the ground.

(a) Draw a diagram which represents the information given.

(b) Calculate, to two significant figures,

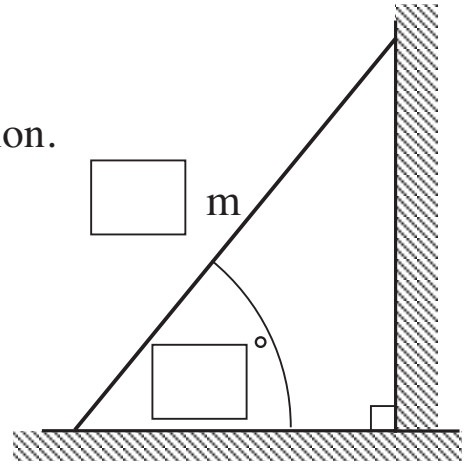
(i) the height the ladder reaches up the wall

(ii) the distance the foot of the ladder is from the wall.

(c) The top of the ladder is lowered so that it reaches 1.75 m up the wall, still touching the wall. Calculate the angle that the ladder now makes with the horizontal.

Solution

(a) Draw a diagram to represent this information.



(b) (i) Height ladder reaches up the wall:

Height = ☐ × sin ☐ °

= ☐ to 2 sig. fig.

(ii) Distance of foot of ladder from the wall:

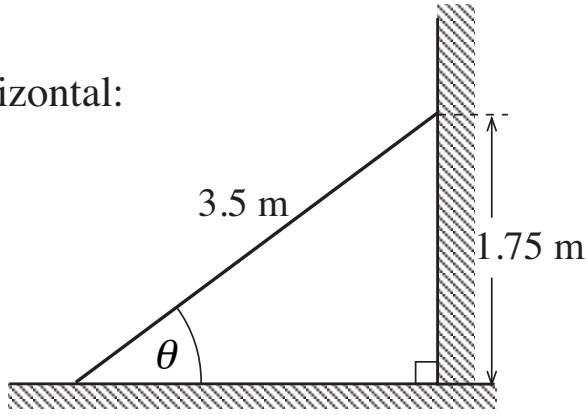
Distance = ☐ × cos ☐ °

= ☐ to 2 sig. fig.

(c) Angle ladder now makes with the horizontal:

$$\sin\theta = \frac{\square}{\square} = \square$$

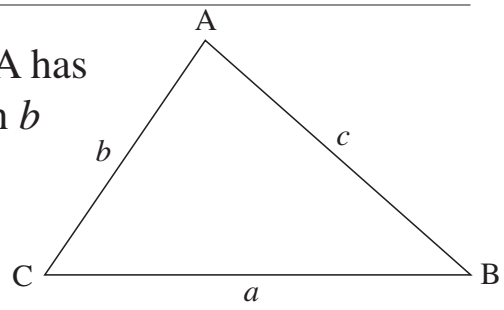
$$\theta = \square\ ^{\circ}$$



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Data Sheet I2.3

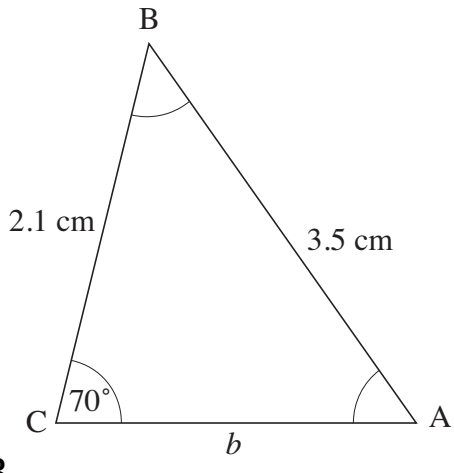
Sine Rule



In the triangle ABC, the side opposite angle A has length a, the side opposite angle B has length b and the side opposite angle C has length c.

The *sine rule* states that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example

Find the unknown angles and the length of the unknown side of this triangle.

Solution

Using the sine rule,

$$\frac{\sin A}{\square} = \frac{\sin \square^{o}}{\square} = \frac{\sin B}{b}$$

Hence $\sin A = \dfrac{\square \times \sin \square^{o}}{\square} = \square$

$A \approx \square^{o}$

As angles in a triangle sum to $\square^{o}$, then angle

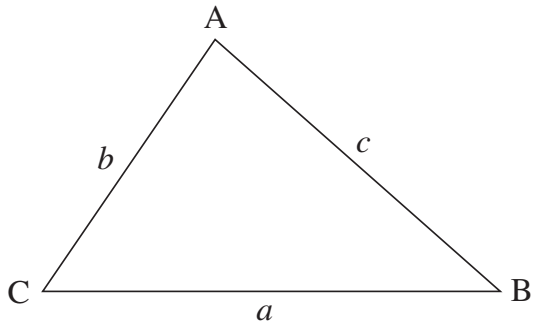
$B = \square^{o} - \square^{o} - A \approx \square^{o}$

so $\dfrac{\sin \square^{o}}{\square} = \dfrac{\sin B}{b}$

$b = \dfrac{\square \sin \square^{o}}{\sin \square} \approx \square$ cm

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Data Sheet I2.4 *Cosine Rule*

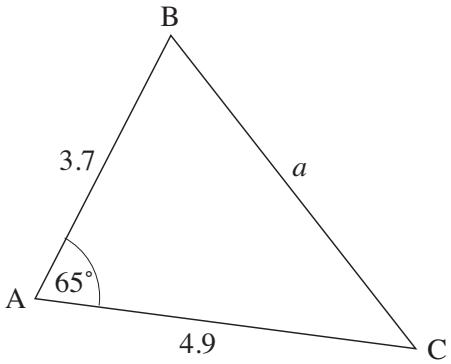


The *cosine rule* states that

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example

Find the length of the unknown side and angles of this triangle.



Solution

Using the cosine rule,

$a^2 = \square^2 + \square^2 - 2 \times \square \times \square \times \cos \square^{\circ}$

$= \square$ So $a = \square$ to 2 decimal places

To find the unknown angles, using the sine rule,

$$\frac{\sin\square^{\circ}}{a} = \frac{\sin B}{\square} = \frac{\sin C}{\square}$$

So $\sin B = \dfrac{\square \times \sin\square^{\circ}}{\square} = \square \Rightarrow B = \square^{\circ}$

Since angles in the triangle add to $\square^{\circ}$, you can calculate angle C by

$C = \square^{\circ} - 65° - \square^{\circ} \approx \square^{\circ}$

Note: You could have calculated angle B using the cosine rule, since

$$b^2 = c^2 + a^2 - 2ca\cos B$$

so $\cos B = \dfrac{3.7^2 + \square^2 - 4.9^2}{2 \times 3.7 \times \square} = \dfrac{\square}{\square} \Rightarrow B = \square^{\circ}$

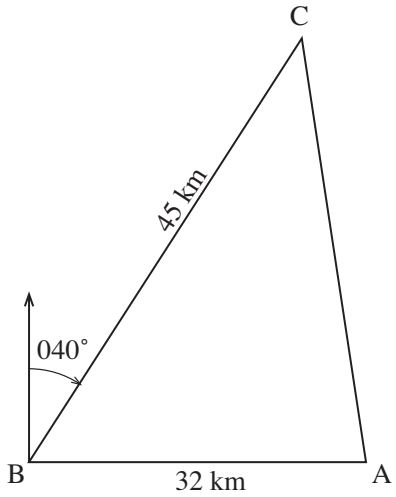
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Data Sheet I2.5 *Problems with Bearings*

The diagram shows the journey of a ship which sailed from Port A to Port B and then to Port C.

Port B is located 32 km due West of Port A.

Port C is 45 km from Port B on a bearing of 040°.



(a) Calculate, to 3 significant figures, the distance AC.

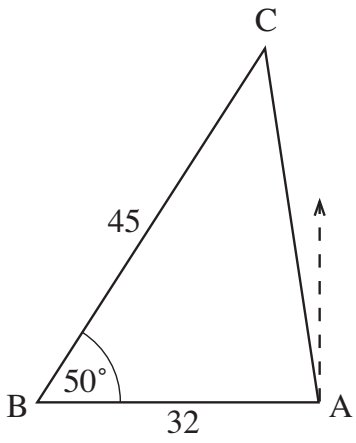
Using the cosine rule,

$$AC^2 = \square^2 + \square^2 - 2 \times \square \times \square \times \cos \square^{o}$$

$$= \square$$

$$AC = \square \text{ to 3 significant figures}$$

(b) Calculate the bearing of Port C from Port A, to 3 significant figures.



The bearing of C from A is

$$270° + \text{angle BAC}$$

Using the sine rule,

$$\frac{\sin BAC}{\square} = \frac{\sin \square^{o}}{\square}$$

$$\sin BAC = \frac{\square \times \sin \square^{o}}{\square} \approx \square$$

Angle BAC $\approx \square^{o}$ to 3 sig. fig. and bearing of Port A from Port B is

$$270° + \square^{o} = \square^{o} \text{ to 3 sig. fig.}$$

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Data Sheet I2.6 *Special Value*

Note that

$$\sin^2\theta + \cos^2\theta = 1$$

for any angle θ.

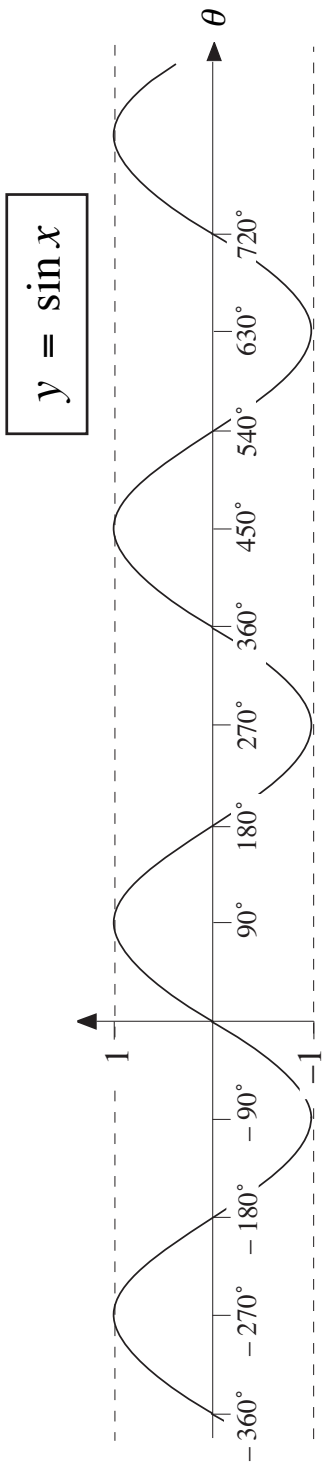
Also, there are some special values for some angles, as shown below.

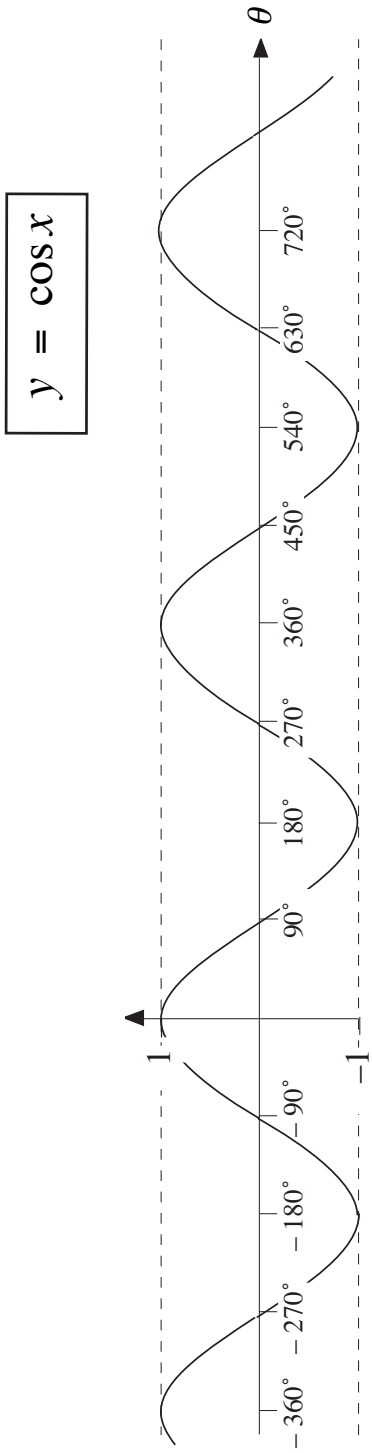
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	infinite

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Data Sheet I2.7

Sine and Cosine Functions

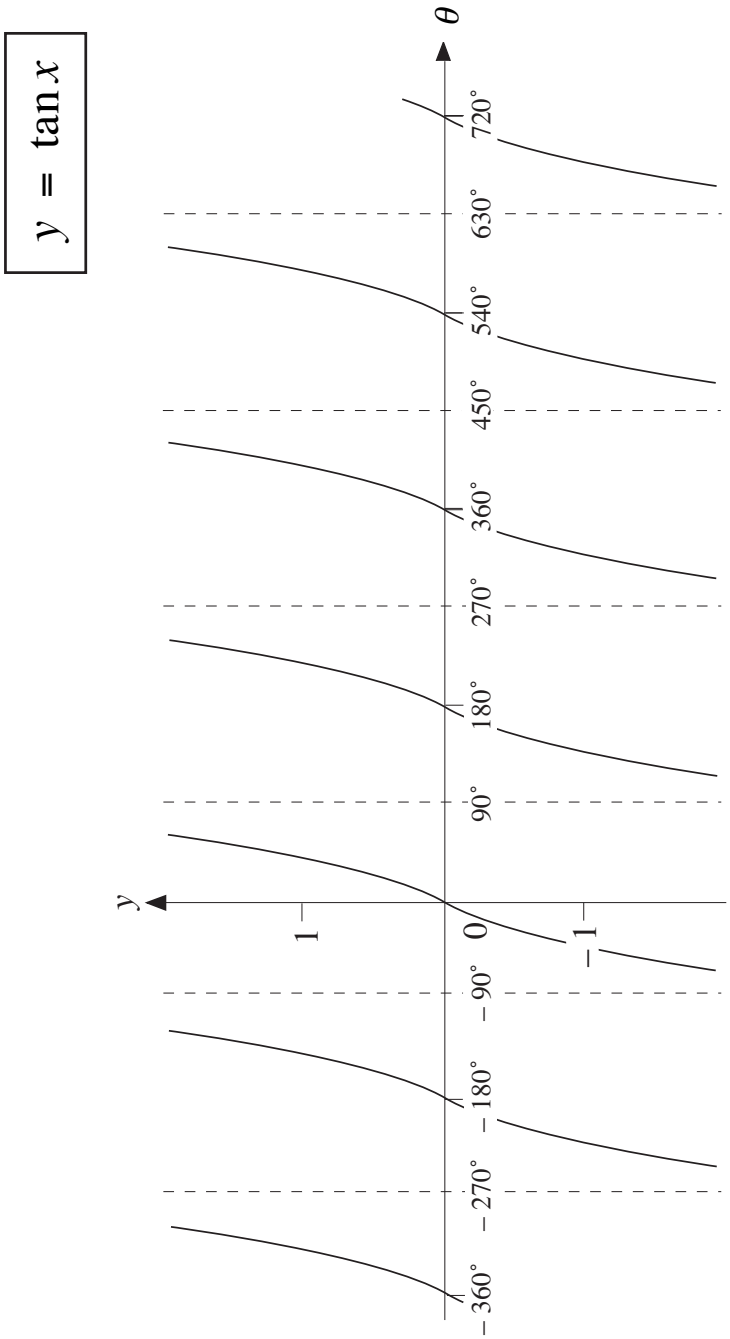




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Data Sheet I2.8

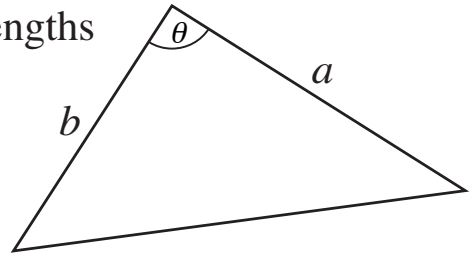
Tangent Functions



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Data Sheet I2.9 *Area of Any Triangle*

Note that the area of any triangle with side lengths a and b and included angle θ, is given by



$$A = \frac{1}{2}ab\sin\theta$$

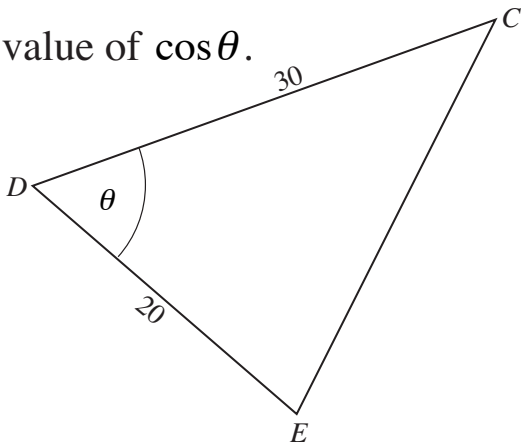
Example

Given that $\sin\theta = \frac{\sqrt{3}}{2}$, $0° \le \theta \le 90°$

(a) Express in fractional or surd form the value of $\cos\theta$.

(b) Show that the area of triangle CDE is $150\sqrt{3}$ square units, where $CD = 30$ units and $DE = 20$ units.

(c) Calculate the side length EC.



Solution

(a) Since $\sin^2\theta + \cos^2\theta = 1$, then

$$\cos^2\theta = 1 - \square^2 = \square$$

and $\cos\theta = \square$

(b) Area $= \frac{1}{2} \times \square \times \square \times \sin\theta$

$= \square\sqrt{3}$

(c) Using the cosine rule,

$$EC^2 = \square^2 + \square^2 - 2 \times \square \times \square \times \cos\theta$$

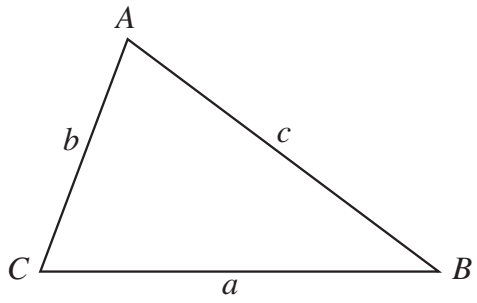
$= \square$

$EC = \square$ units

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Data Sheet I2.10 *Heron's Formula*

Heron's formula is useful for calculating the area, A, of a triangle if you know the lengths of all three sides.



Heron's formula states that

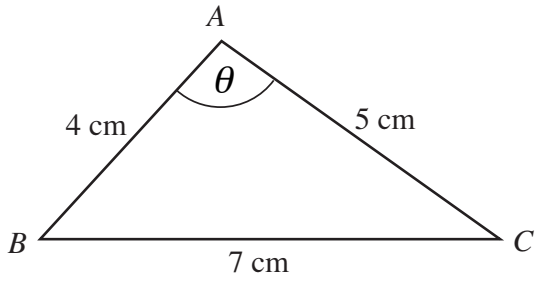
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is half the perimeter of the triangle, i.e. $s = \dfrac{a+b+c}{2}$.

Example

For the triangle shown, find

(a) the area of the triangle, (b) angle θ.



Solution

(a) Using Heron's formula,

$$s = \frac{1}{2}(\square + \square + \square) \text{ cm} = \square \text{ cm}$$

$$\text{So} \quad A = \sqrt{\square(\square - 7)(\square - 5)(\square - 4)}$$

$$= \sqrt{\square \times \square \times \square \times \square}$$

$$= \sqrt{\square}$$

$$\approx \square \text{ cm}^2$$

(b) Using the formula, $A = \dfrac{1}{2}bc\sin\theta$

$$\sin\theta = \frac{2 \times A}{b \times c} = \frac{2 \times \square}{\square \times \square} = \square$$

Hence $\theta \approx \square^{\circ}$

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